



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



PROBLEMS UNDER FOURIER SINE AND COSINE TRANSFORMS:

$$F_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin sx \, dx = F_s(s)$$

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} F_s(s) \sin sx \, ds$$

$$F_c[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos sx \, ds = F_c(s)$$

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} F_c(s) \cos sx \, ds$$

① Find the Fourier cosine transform for

$$f(x) = \begin{cases} x & ; 0 < x < 1 \\ 2-x & ; 1 < x < 2 \\ 0 & ; x > 2 \end{cases}$$

Solution:

The Fourier cosine transform for $f(x)$ is

$$F_c[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos sx \, dx$$

$$\begin{array}{ccc} f(x)=x & f(x)=2-x & f(x)=0 \\ \begin{array}{c} 0 \quad 1 \end{array} & \begin{array}{c} 1 \quad 2 \end{array} & \begin{array}{c} 2 \quad \infty \end{array} \end{array}$$

$$= \sqrt{\frac{2}{\pi}} \left[\int_0^1 x \cos sx \, dx + \int_1^2 (2-x) \cos sx \, dx \right]$$



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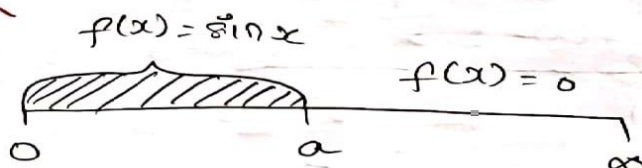
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$$\begin{aligned} & + \int_0^{\infty} \cos x \, dx \Big] \\ &= \sqrt{\frac{2}{\pi}} \left[x \left(\frac{\sin sx}{s} \right) - (1) \left(-\frac{\cos sx}{s^2} \right) \right]_0^1 \\ & \quad + \left[(2-x) \left(\frac{\sin sx}{s} \right) - (1) \left(-\frac{\cos sx}{s^2} \right) \right]_1^2 \\ &= \sqrt{\frac{2}{\pi}} \left[\frac{\sin s}{s} + \frac{\cos s}{s^2} - 0 - \frac{1}{s^2} - \frac{\cos 2s}{s^2} - \frac{\sin s}{s} + \frac{\cos s}{s^2} \right] \\ &= \sqrt{\frac{2}{\pi}} \left[\frac{\cos s}{s^2} - \frac{1}{s^2} - \frac{\cos 2s}{s^2} + \frac{\cos s}{s^2} \right] \\ & \boxed{F_c[f(x)] = \sqrt{\frac{2}{\pi}} \left(\frac{2 \cos s - \cos 2s - 1}{s^2} \right)} \end{aligned}$$

② Find the Fourier sine transform for
$$f(x) = \begin{cases} \sin x & 0 < x < a \\ 0 & x > a \end{cases}$$

Solution:



The Fourier sine transform for $f(x)$ is



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$$\begin{aligned}F_s[f(x)] &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin sx \, dx \\&= \sqrt{\frac{2}{\pi}} \left[\int_0^a \sin x \sin sx \, dx + \int_0^{\infty} 0 \sin sx \, dx \right] \\&= \sqrt{\frac{2}{\pi}} \int_0^a \sin x \sin sx \, dx\end{aligned}$$

$$\because \sin A \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{1}{2} \int_0^a [\cos(1-s)x - \cos(1+s)x] \, dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{\sin(1-s)x}{1-s} - \frac{\sin(1+s)x}{1+s} \right]_0^a$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{\sin(1-s)a}{1-s} - \frac{\sin(1+s)a}{1+s} \right]$$