



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



UNIT 4 Fourier Series and Fourier Transform

Unit - III FOURIER SERIES

PERIODIC FUNCTION:

A function $f(x)$ is said to be periodic if for all x , $f(x+T) = f(x)$, where T is a positive constant. The least value of $T > 0$ is called the period of $f(x)$.

For example:-

$$f(x) = \sin x = \sin(x + 2\pi) = \dots$$

$\therefore \sin x$ is a periodic function with period 2π .

RESULTS:

$$\sin 0 = 0$$

$$\sin n\pi = 0$$

$$\sin 2n\pi = 0$$

$$\cos 0 = 1$$

$$\cos n\pi = (-1)^n$$

$$\cos 2n\pi = 1$$

$$\cos(1+n)\pi = -(-1)^n$$

$$\cos(1-n)\pi = -(-1)^n$$

$$\cos \frac{n\pi}{2} = 0; n=3, \dots$$

FOURIER - EULER FORMULA:

The Fourier series for the function $f(x)$ defined in the interval $(c, c+2\pi)$ with period 2π is given by.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

where,

$$a_0 = \frac{1}{\pi} \int_c^{c+2\pi} f(x) dx \quad \text{--- (1)}$$



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$$a_n = \frac{1}{\pi} \int_c^{c+2\pi} f(x) \cos nx \, dx \quad - (2)$$

$$b_n = \frac{1}{\pi} \int_c^{c+2\pi} f(x) \sin nx \, dx \quad - (3)$$

The values of a_0 , a_n , b_n are known as Fourier-Euler formulae.

DIRICHLET'S CONDITIONS FOR THE EXPANSION OF FOURIER SERIES.

Any function $f(x)$ can be expressed as a Fourier series in $(c, c+2\pi)$ if

- i) $f(x)$ must be periodic in $(c, c+2\pi)$.
- ii) $f(x)$ must be single valued and finite in $(c, c+2\pi)$.
- iii) $f(x)$ has finite number of maxima and minima in $(c, c+2\pi)$.
- iv) $f(x)$ has finite number of finite discontinuities in $(c, c+2\pi)$.