3. Stefan Boltzman law.

The emissive power of black surface can be found by integrating the expression for Plank's law over all wavelengths. Thus.

$$e_{b\lambda} = \int_{\lambda=0}^{\lambda=\infty} e_{b\lambda} \cdot d\lambda = \int_{\lambda=0}^{\lambda=\infty} \frac{2\pi C_1}{\lambda^5 \left[e^{(C_2/\lambda T)} - 1 \right]}$$

Let $x = 1/\lambda$ $\therefore dx = (-1/x^2) d\lambda$.

$$\therefore e_b = 2\pi C_1 \int_0^\infty \frac{x^2 \cdot dx}{\left[e^{(C_2 x/T)} - 1 \right]}$$

$$= 2\pi C_1 \int_0^\infty x^2 \left[e^{(C_2 x/T)} - 1 \right]^{-1} dx.$$

$$= 2\pi C_1 \cdot \int_0^\infty x^3 \left[e^{-(C_2/T)x} + e^{-(C_2/T)2x} + \dots \right] dx$$

$$= 2\pi C_1 \frac{6T^4}{C_2^4} \left(1 + \frac{1}{2^4} + \frac{1}{3^4} + \dots \right)$$

Try @ home.

$$= 2\pi C_1 \frac{6T^4}{C_2^4}\left(\frac{\pi^4}{90}\right)$$

$e_b \propto T^4$

$e_b = \sigma T^4$. $\quad \sigma = \frac{2\pi C_1 \times 6 \pi^4}{C_2^4 \times 90}$.

σ = Stefan Boltzman constant

$\sigma = 5.67 \times 10^{-8}$ $W/m^2 K^4$.

For gray body.

$e = \epsilon \cdot e_b = \epsilon \sigma T^4$ $\quad$ T = Absolute temp

4) Kirchoff's Law:- ($\epsilon_\lambda = \alpha_\lambda$) and ($\epsilon = \alpha$)

The [monochromatic] emissivity of a surface is equal to the [monochromatic] absorptivity of the surface (emitted in a diffuse manner). at given temp. T

Proof:-> $\epsilon = \frac{e}{e_b} = \frac{\int_{\lambda=0}^{\infty} e_\lambda \cdot d\lambda}{e_b} = \frac{\int_0^\infty \epsilon_\lambda \cdot e_{b\lambda} \cdot d\lambda}{e_b}$

$\left[\because \epsilon_\lambda = \frac{e_\lambda}{e_{b\lambda}}\right]$ $\quad \therefore \epsilon = \epsilon_\lambda \cdot \frac{\left(\int_0^\infty e_{b\lambda} \cdot d\lambda\right)}{e_b}$ $\quad \epsilon_\lambda$ = constant.

$\epsilon = \epsilon_\lambda \longrightarrow (1)$

For black body (unity)

Also $\alpha = \frac{H_a}{H} = \frac{\int_0^\infty H_{a\lambda}\, d\lambda}{H_b}$ $\left[\because \epsilon_\lambda = \frac{H_{a\lambda}}{H_\lambda}\right]$

Emissive power [Irradiation]

$\alpha_\lambda = \frac{H_{a\lambda}}{H_\lambda}$

$$\alpha = \frac{\int_0^\infty \epsilon_\lambda H_\lambda\, d\lambda}{H_b}$$

$$\alpha = \epsilon_\lambda \frac{\int_0^\infty H_{b\lambda}\, d\lambda}{H_b}$$

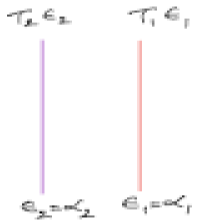
$$\alpha = \epsilon_\lambda \frac{H_b}{H_b}$$

$$\alpha = \epsilon_\lambda \rightarrow (2)$$

From (1) and (2) $\alpha = \epsilon = \epsilon_\lambda \rightarrow (I)$

(*) So a surface for which both the equation hold is called a diffuse-gray surface. Non-black surfaces are diffuse-gray.

For blackbody $\alpha = 1$, $\epsilon = 1$, $\epsilon_1 = \epsilon_2 = \epsilon$, $\alpha_1 = \alpha_2 = \alpha$. $T_1 > T_2$.



(*) $e_1 = \epsilon_1 \sigma T_1^4 \rightarrow (1)$

$e_2 = \epsilon_2 \sigma T_2^4 \rightarrow (2)$

$e_1 = \alpha_1 \sigma T_1^4$ & $e_2 = \alpha_2 \sigma T_2^4$

$e_1 - e_2 = \alpha\sigma(T_1^4 - T_2^4) = \epsilon\sigma(T_1^4 - T_2^4)$

Net radiation.

$T_1 = T_2 \longrightarrow \alpha = \epsilon$.

References:

1. Kothandaraman C.P “Fundamentals of Heat and Mass Transfer” New Age International, New Delhi,4th Edition 2012 (Unit I, II, III, IV, V).
2. Frank P. Incropera and David P. DeWitt, “Fundamentals of Heat and Mass Transfer”, John Wiley and Sons, New Jersey,6th Edition1998(Unit I,II,III,IV, V)
3. MIT open courseware – https://ocw.mit.edu/courses/mechanical-engineering

Other web sources.