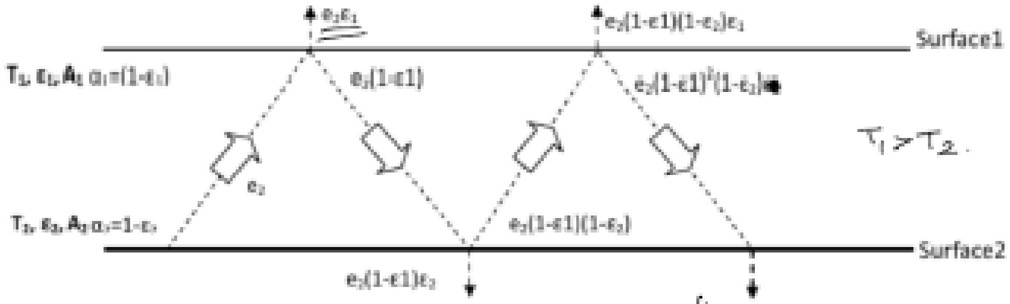
Topic - Gray body radiation

Radiation heat exchange between two parallel infinite Gray surfaces.



$e_1 = \epsilon_1 \sigma T_1^4 ; \quad e_2 = \epsilon_2 \sigma T_2^4$

Radiant heat flow emitted by ② & reading ①

$$q_{2\to1} = e_2\epsilon_1 + e_2(1-\epsilon_1)(1-\epsilon_2) + e_2(1-\epsilon_1)^2(1-\epsilon_2)^2 + \dots \rightarrow (1)$$

The R.H.S term is a geometric progression,

$$q_{2\to1} = e_2\epsilon_1 \left[\frac{1}{1-(1-\epsilon_1)(1-\epsilon_2)}\right]$$

$$q_{2\to1} = e_2 \left[\frac{\epsilon_1}{1-[1-\epsilon_2-\epsilon_1+\epsilon_1\epsilon_2]}\right]$$

$$q_{2\to1} = e_2 \left[\frac{\epsilon_1}{\epsilon_1+\epsilon_2-\epsilon_1\epsilon_2}\right] \rightarrow (2)$$

$$q_{1\to2} = e_1 \left[\frac{\epsilon_2}{\epsilon_1+\epsilon_2-\epsilon_1\epsilon_2}\right]$$

Radiant heat flux emitted by ① & reaching ②

$$q_{1\to2} = e_1 \epsilon_2 + e_1(1-\epsilon_1)(1-\epsilon_2) + e_1(1-\epsilon_1)^2(1-\epsilon_2)^2 + \ldots \rightarrow ③$$

The R.H.S term is a geometric progression,

$$q_{1\to2} = e_1 \epsilon_2 \left[\frac{1}{1-(1-\epsilon_1)(1-\epsilon_2)}\right]$$

$$q_{1\to2} = e_1 \left[\frac{\epsilon_2}{1-[1-\epsilon_2-\epsilon_1+\epsilon_1\epsilon_2]}\right]$$

$$q_{1\to2} = e_1 \left[\frac{\epsilon_2}{\epsilon_1+\epsilon_2-\epsilon_1\epsilon_2}\right] \rightarrow ④$$

If $T_1 > T_2$, Net radiation between ① & ②

$$q_{1-2} - q_{2-1} = q_{net} = \frac{e_1\epsilon_2 - e_2\epsilon_1}{\epsilon_1+\epsilon_2-\epsilon_1\epsilon_2} \rightarrow ⑤$$

$e_1 = \epsilon_1 \sigma T_1^4$ & $e_2 = \epsilon_2 \sigma T_2^4$ ⊗ → Stefan Boltzman law.

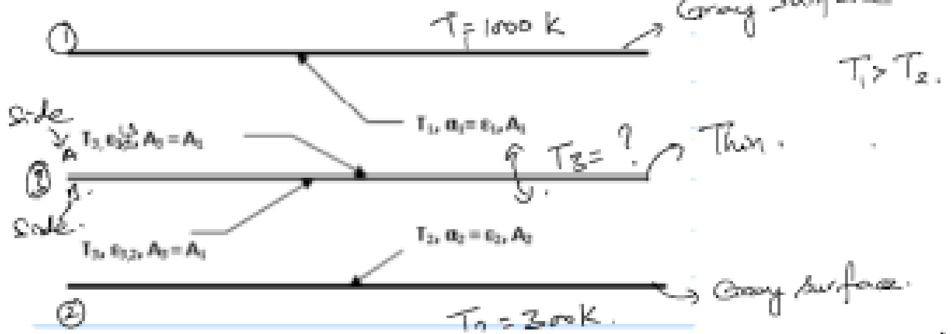
$$\therefore q_{net} = \frac{\epsilon_1 \sigma T_1^4 \epsilon_2 - \epsilon_2 \sigma T_2^4 \epsilon_1}{\epsilon_1+\epsilon_2-\epsilon_1\epsilon_2}$$

$$q_{net} = \frac{\epsilon_1\epsilon_2\sigma(T_1^4 - T_2^4)}{\epsilon_1+\epsilon_2-\epsilon_1\epsilon_2}$$

$\div\ \epsilon_1\epsilon_2$

$$q_{net} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\epsilon_2} + \frac{1}{\epsilon_1} - 1\right)} \rightarrow (6)$$

Radiation shields.



It is possible to reduce the net radiation heat exchange between two parallel gray surfaces by a third surface in between them.

T_3 = Radiation shield temperature.

ϵ_3 = Emissivity of radiation shield.

$$q_{1-3} = \frac{\sigma(T_1^4 - T_3^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_{1,3}} - 1\right)} \rightarrow (1)$$

$$q_{3-2} = \frac{\sigma(T_3^4 - T_2^4)}{\left(\frac{1}{\epsilon_{3,2}} + \frac{1}{\epsilon_2} - 1\right)} \rightarrow (2)$$

For steady-state condition, these two must be equal.

$$\therefore \quad q_{1-3} = q_{3-2} \quad \text{[Energy balance]}$$

$$\frac{\sigma(T_1^4 - T_3^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_{13}} - 1\right)} = \frac{\sigma(T_3^4 - T_2^4)}{\left(\frac{1}{\epsilon_{32}} + \frac{1}{\epsilon_2} - 1\right)}$$

Let $X = \frac{1}{\epsilon_1} + \frac{1}{\epsilon_{13}} - 1$ & $Y = \frac{1}{\epsilon_{32}} + \frac{1}{\epsilon_2} - 1$.

$$\therefore \quad \frac{(T_1^4 - T_3^4)}{X} = \frac{(T_3^4 - T_2^4)}{Y}$$

Solve for T_3,

$$(T_1^4 - T_3^4)\,Y = X(T_3^4 - T_2^4)$$

$$Y T_1^4 - Y T_3^4 = X T_3^4 - X T_2^4$$

$$Y T_1^4 + X T_2^4 = T_3^4 (X + Y).$$

$$\therefore \quad T_3^4 = \frac{Y T_1^4 + X T_2^4}{(X+Y)}$$

$$T_3 = \left[\frac{T_1^4 + \frac{X}{Y} T_2^4}{1 + \frac{X}{Y}}\right]^{1/4} \rightarrow (3)$$

Special case:

(1) $\epsilon_1 = \epsilon_2 = \epsilon_{31} = \epsilon_{32} = \epsilon$, then $X = Y = \frac{2}{\epsilon} - 1$.

then $T_3 = \left[\frac{T_1^4 + T_2^4}{2}\right]^{1/4} \rightarrow (4)$

$$q_{rad\,shield} = \sigma \left[\frac{T_1^4 - \left(\frac{T_1^4 + T_2^4}{2}\right)}{\frac{2}{\epsilon} - 1}\right]$$

$$= \frac{\sigma\left[T_1^4 - T_2^4\right]}{2\left(\frac{2}{\epsilon} - 1\right)} \rightarrow (5)$$

Comment:

(1) When emissivities of all surfaces are equal, the net radiation reduces by 50%.

(2) If there are 'N' radiation shields then.

$$q_{1-2\,N\text{-shields}} = \frac{1}{(N+1)}\, q_{1-2\,\text{without shield}}.$$

References:

1. Kothandaraman C.P "Fundamentals of Heat and Mass Transfer" New Age International, New Delhi,4th Edition 2012 (Unit I, II, III, IV, V).
2. Frank P. Incropera and David P. DeWitt, "Fundamentals of Heat and Mass Transfer", John Wiley and Sons, New Jersey,6th Edition1998(Unit I,II,III,IV, V)
3. MIT open courseware – https://ocw.mit.edu/courses/mechanical-engineering

Other web sources