Steady-State Molecular Diffusion

This part is an application to the general differential equation of mass transfer. The objective is to solve the differential equation of mass transfer under steady state conditions at different conditions (chemical reaction, one dimensional or more etc.).

Two approaches will be used to simplify the differential equations of mass transfer:

1. Fick's equation and the general differential equation for mass transfer can be simplified by eliminating the terms that do not apply to the physical situation.
2. A material balance can be performed on a differential volume element of the control volume for mass transfer.

One dimensional mass transfer independent of chemical reaction

The diffusion coefficient or mass diffusivity for a gas may be experimentally measured in an Arnold diffusion cell. This cell is illustrated schematically in Figure 1. The narrow tube, which is partially filled with pure liquid A, is maintained at a constant temperature and pressure. Gas B, which flows across the open end of the tube, has a negligible solubility in liquid A and is also chemically inert to A. Component A vaporizes and diffuses into the gas phase; the rate of vaporization may be physically measured and may also be mathematically expressed in terms of the molar mass flux.

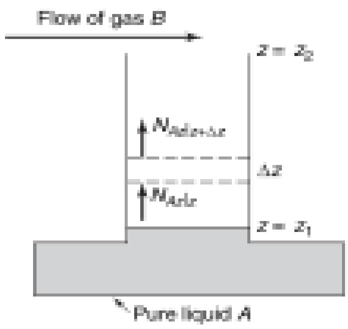

Fig. 1, Arnold diffusion cell

Required: to get the flux and concentration profile for this system.

Assumptions:

1. Steady state conditions
2. Unidirectional mass transfer in z direction
3. No chemical reaction occur

The general differential equation for mass transfer is given by:

$$\nabla.\vec{N}_A + \frac{\partial c_A}{\partial t} - R_A = 0$$

In rectangular coordinates this equation is:

$$\frac{\partial}{\partial x}N_{A,x} + \frac{\partial}{\partial y}N_{A,y} + \frac{\partial}{\partial z}N_{A,z} + \frac{\partial c_A}{\partial t} - R_A = 0$$

Apply the above assumptions to this equation:

$$\therefore \frac{d}{dz}N_{A,z} = 0$$

It means that the molar flux of A is constant over the entire diffusion path from z_1 to z_2.

The molar flux is defined by the equation:

$$N_A = -cD_{AB}\frac{dy_A}{dz} + y_A(N_A + N_B)$$

According to the conditions of the system (B is insoluble and chemically inert to A)

$$\therefore N_B = 0$$

$$N_A = -cD_{AB}\frac{dy_A}{dz} + y_A N_A$$

$$N_A = -\frac{cD_{AB}}{1 - y_A}\frac{dy_A}{dz}$$

To get the flux N_A the above equation has to be integrated between z_1 and z_2 by using the boundary conditions:

at $z = z_1$ $\qquad$ $y_A = y_{A1}$

at $z = z_2$ $\qquad$ $y_A = y_{A2}$

$$N_A \int_{z_1}^{z_2} dz = cD_{AB} \int_{y_{A1}}^{y_{A2}} -\frac{dy_A}{1-y_A}$$

$$N_A = \frac{cD_{AB}}{z_2 - z_1} \ln \frac{(1-y_{A2})}{(1-y_{A1})} \qquad (1)$$

The above equation (equation 1) is commonly referred to as equations for steady-state diffusion of one gas through a second non diffusing gas or stagnant gas.

Absorption and humidification are typical operations defined by this two equation.

Some important notes:

Concentration for gas phase:	
Total concentration: $c = \frac{P}{RT}$	Concentration of A: $c_A = \frac{p_A}{RT}$
$y_A = \frac{p_A}{P}$	

Example 1:

References:

1. Kothandaraman C.P "Fundamentals of Heat and Mass Transfer" New Age International, New Delhi,4th Edition 2012 (Unit I, II, III, IV, V).
2. Frank P. Incropera and David P. DeWitt, "Fundamentals of Heat and Mass Transfer", John Wiley and Sons, New Jersey,6th Edition1998(Unit I,II,III,IV, V)
3. MIT open courseware – https://ocw.mit.edu/courses/mechanical-engineering

Other web sources