Topic - Tutorial- Fick's Law of diffusion

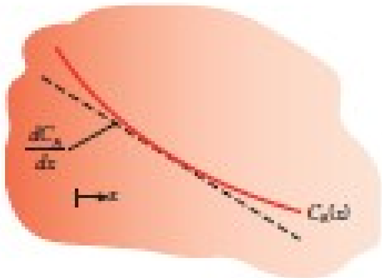
Consider the diffusion of hydrogen (species A) in air, liquid water, or iron (species B) at $T = 293$ K. Calculate the species flux on both molar and mass bases if the concentration gradient at a particular location is $dC_A/dx = 1\ \text{kmol/m}^3 \cdot \text{m}$. Compare the value of the mass diffusivity to the thermal diffusivity. The mole fraction of the hydrogen, $x_A$, is much less than unity.

## Solution

***Known:*** Concentration gradient of hydrogen in air, liquid water, or iron at $T = 293$ K.

***Find:*** Molar and mass fluxes of hydrogen and the relative values of the mass and thermal diffusivities for the three cases.

***Schematic:***




***Assumptions:*** Steady-state conditions.

***Properties:*** Table A.8, hydrogen–air (298 K): $D_{AB} = 0.41 \times 10^{-4}\ \text{m}^2/\text{s}$, hydrogen–water (298 K): $D_{AB} = 0.63 \times 10^{-8}\ \text{m}^2/\text{s}$, hydrogen–iron (293 K): $D_{AB} = 0.26 \times 10^{-12}\ \text{m}^2/\text{s}$. Table A.4, air (293 K): $\alpha = 21.6 \times 10^{-6}\ \text{m}^2/\text{s}$; Table A.6, water (293 K): $k = 0.603\ \text{W/m} \cdot \text{K}$, $\rho = 998\ \text{kg/m}^3$, $c_p = 4182\ \text{J/k} \cdot \text{K}$. Table A.1, iron (300 K): $\alpha = 23.1 \times 10^{-6}\ \text{m}^2/\text{s}$.

***Analysis:*** Using Equation 14.14, we find that the mass diffusivity of hydrogen in air at $T = 293$ K is

$$D_{AB,T} = D_{AB,298\,K} \times \left(\frac{T}{298\ \text{K}}\right)^{3/2} = 0.41 \times 10^{-4}\ \text{m}^2/\text{s} \times \left(\frac{293\ \text{K}}{298\ \text{K}}\right)^{3/2} = 0.40 \times 10^{-4}\ \text{m}^2/\text{s}$$

For the case where hydrogen is a *dilute species*, that is, $x_A \ll 1$, the thermal properties of the medium can be taken to be those of the *host medium*, species B. The thermal diffusivity of water is

$$\alpha = \frac{k}{\rho c_p} = \frac{0.603\ \text{W/m} \cdot \text{K}}{998\ \text{kg/m}^3 \cdot 4182\ \text{J/kg} \cdot \text{K}} = 0.144 \times 10^{-6}\ \text{m}^2/\text{s}$$

The ratio of the thermal diffusivity to the mass diffusivity is the Lewis number, $Le$, defined in Equation 6.50.

The molar flux of hydrogen is described by Fick's law, Equation 14.13,

$$J_A^* = -CD_{AB}\frac{dx_A}{dx}$$

The total molar concentration, $C$, is approximately constant since A is a dilute species; therefore

$$J_A^* = -D_{AB}\frac{dC_A}{dx}$$

Hence, for the hydrogen–air mixture,

$$J_A^* = -0.40 \times 10^{-4}\ \mathrm{m^2/s} \times 1\ \frac{\mathrm{kmol}}{\mathrm{m^3 \cdot m}} = -4 \times 10^{-5}\ \frac{\mathrm{kmol}}{\mathrm{s \cdot m^2}} \quad \triangleleft$$

The mass flux of hydrogen in air is found from the expression

$$j_A = \mathcal{M}_A J_A^* = 2\ \frac{\mathrm{kg}}{\mathrm{kmol}} \times \left(-4 \times 10^{-5}\ \frac{\mathrm{kmol}}{\mathrm{s \cdot m^2}}\right) = -8 \times 10^{-5}\ \frac{\mathrm{kg}}{\mathrm{s \cdot m^2}} \quad \triangleleft$$

The results for the three different mixtures are summarized in the following table. $\triangleleft$

| Species B | $\alpha \times 10^6$ (m²/s) | $D_{AB} \times 10^6$ (m²/s) | $Le$ | $j_A \times 10^6$ (kg/s·m²) |
|---|---|---|---|---|
| Air | 21.6 | 40 | 0.54 | 80 |
| Water | 0.14 | $6.3 \times 10^{-3}$ | 23 | $13 \times 10^{-3}$ |
| Iron | 23.1 | $260 \times 10^{-9}$ | $89 \times 10^6$ | $0.52 \times 10^{-6}$ |

*Comments:*

1. The thermal diffusivities of the three media vary by two orders of magnitude. We saw in Chapter 5 that this relatively broad range of thermal diffusivities is responsible for the different rates at which objects respond thermally during transient conduction processes. Mass diffusivities can vary by 8 or more orders of magnitude, with the highest diffusivities associated with diffusion in gases and the lowest diffusivities associated with diffusion in solids. Different materials respond to mass transfer at very different rates, depending on whether the host medium is a gas, liquid, or solid.
2. The ratio of the thermal diffusivity to the mass diffusivity, the Lewis number, is typically of order unity for gases. This implies that changes in the thermal and species distributions progress at similar rates in gases that undergo simultaneous heat and mass transfer by diffusion. In solids or liquids, thermal energy is conducted much more readily than chemical species can be transferred by diffusion.

*In order to avoid pressure build up ammonia gas at atmospheric pressure in a pipe is vented to atmosphere through a pipe of 3 mm dia and 20 m length. Determine the mass of ammonia diffusing out and mass of air diffusing in per hour.* Assume $D = 0.28 \times 10^{-4}$ m$^2$/s, $M = 17$ kg/kg mole

**Solution:** $P_{NH_3}$ in pipe = 1 atm.

$P_{NH_3}$ at the outlet = 0

$$m_{NH_3} = \frac{D.A.}{\Re T} \frac{P_{NH_3 1} - P_{NH_3 2}}{L} \times M$$

$$= 0.28 \times 10^{-4} \times \frac{\pi}{4}(0.003)^2 \times \frac{(1.013 \times 10^5 - 0)}{20} \times 3600 \times 17/8315$$

$$= \mathbf{7.38 \times 10^{-6}\ kg/hr.}$$

$$m_{air},\ N_B = -N_A = -7.38 \times 10^{-6} \times 28.97/17$$

$$= \mathbf{-1.26 \times 10^{-5}\ kg/hr.}$$

$$M_{air} = 28.97 \text{ kg/kg mole.}$$

**References:**

1. Kothandaraman C.P "Fundamentals of Heat and Mass Transfer" New Age International, New Delhi,4th Edition 2012 (Unit I, II, III, IV, V).
2. Frank P. Incropera and David P. DeWitt, "Fundamentals of Heat and Mass Transfer", John Wiley and Sons, New Jersey,6th Edition1998(Unit I,II,III,IV, V)
3. MIT open courseware – https://ocw.mit.edu/courses/mechanical-engineering

Other web sources