Topic - Convective Mass Transfer –Pipe flow

In a mass transfer spray column, a liquid is sprayed into a gas stream, and mass is transferred between the liquid and gas phases. The mass of the drops that are formed from a spray nozzle is considered a function of the nozzle diameter, acceleration of gravity, and surface tension of the liquid against the gas, fluid density, fluid viscosity, fluid velocity, and the viscosity and density of the gas medium. Arrange these variables in dimensionless groups. Should any other variables have been included?

Note: surface tension has a unit of force per unit length or

Solution:

1. Identify the important variables in the system

The important variables are: $m, d, g, \sigma, \rho_L, \mu_L, V, \rho_g, \mu_g$

2. List all the problem variables and parameters, along with their dimensions

Quantity	Dimensional expression
m	M
d	L
g	L/t^2
σ	M/t^2
ρ_L	M/L^3
μ_L	M/Lt
V	L/t
ρ_g	M/L^3
μ_g	M/Lt

The number of dimensionless groups = number of variables – number of fundamental dimensions

The number of dimensionless groups = 9 – 3 = 6

3. Choose a set of reference variables

In this case the reference variables will be: d, V and ρ

$$d = L$$
$$V = L/t$$
$$\rho_L = M/L^3$$

4. Solve the dimensional equations for the dimensions (L, m and t) in terms of the reference variables

$$L = d$$
$$t = d/V$$
$$M = \rho d^3$$

5. Write the dimensional equations for each of the remaining variables in terms of the reference variables

Quantity	Dimensional expression	In terms of the reference variables
m	M	$\rho_L d^3$
g	L/t²	$\frac{V^2}{d}$
σ	M/t²	$\rho_L d V^2$
μ_L	M/Lt	$\rho_L d V$
ρ_g	M/L³	ρ_L
μ_g	M/Lt	$\rho_L d V$

6. The resulting equations are each a dimensional identity, so dividing one side by the other results in one dimensionless group from each equation

N_1	$\left(\frac{\rho_L d^3}{m}\right)$
N_2	$\left(\frac{V^2}{gd}\right)$
N_3	$\left(\frac{\rho_L d V^2}{\sigma}\right)$
N_4	$\left(\frac{\rho_L d V}{\mu_L}\right)$
N_5	$\left(\frac{\rho_L}{\rho_g}\right)$
N_6	$\left(\frac{\rho_L d V}{\mu_g}\right)$

Dry air, flowing at a velocity of 1.5 m/s, enters a 6-m- long, 0.15-m-diameter tube at 310 K and 1.013×10^5 Pa. The inner surface of the tube is lined with a felt material (diameter-to- roughness ratio, $\frac{D}{\varepsilon}$, of 10,000) that is continuously saturated with water at 290 K. Assuming constant temperature of the air and the pipe wall, determine the exit concentration of water in the exit gas stream.

Given that:

Diffusivity of water in air at 300 K = $2.6 \times 10^{-5} \frac{m^2}{s}$

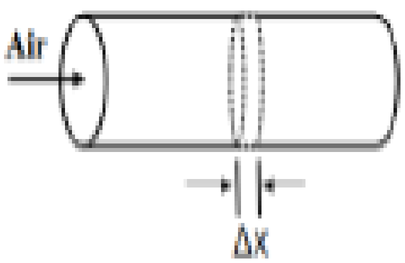
Kinematic viscosity of air at 300 K = $1.569 \times 10^{-5} \frac{m^2}{s}$

The vapor pressure of water at 290 K = 17.5 mm Hg

The gas constant = $0.08206 \frac{L\cdot atm}{mol\cdot K}$

The Fanning friction factor is given by: $f = 0.00791\ Re^{0.12}$

Solution:



By making material balance on water over a control volume of air of a thickness Δx

Water input with air + water transferred to air from the wall by convection = water out with air

$$c_A V \frac{\pi d^2}{4} \Big|_x + k_c (c_{A_s} - c_A) \pi d \Delta x = c_A V \frac{\pi d^2}{4} \Big|_{x+\Delta x}$$

$$\div \frac{\pi d^2}{4} \Delta x$$

$$\frac{c_A \big|_{x+\Delta x} - c_A \big|_x}{\Delta x} = \frac{4}{d} \frac{k_c}{V} (c_{A_s} - c_A)$$

Take the limits as Δx approaches zero

$$\frac{dc_A}{dx} = \frac{4}{d} \frac{k_c}{V} (c_{A_s} - c_A)$$

$$-\int_{c_{A_o}}^{c_{A_L}} \frac{-dc_A}{(c_{A_s} - c_A)} = \frac{4}{d} \frac{k_c}{V} \int_0^L dx$$

$ln\left(\frac{c_{A_s} - c_{A_o}}{c_{A_s} - c_{A_L}}\right) = \frac{4}{d} \frac{k_c}{V} L$	(1)

$$ln\left(\frac{c_{A_s} - c_{A_o}}{c_{A_s} - c_{A_L}}\right) = \frac{4}{d} \frac{k_c}{V} L$$

$d = 0.15\ m$	$V = 1.5 \frac{m}{s}$
$L = 6\ m$	$c_{A_o} = 0 (dry\ air)$
$c_{A_s} = \frac{p_v}{RT} = \frac{17.5}{0.08206 \times 290} = 0.735\ mol/L$	k_c =??? (from anlogy)

From Chilton – Colburn analogy:

$$\frac{k_c}{V_\infty} Sc^{2/3} = \frac{f}{2}$$

$$f = 0.00791\ Re^{0.12}$$

$$Re = \frac{Vd}{\nu} = \frac{1.5 \times 0.15}{1.569 \times 10^{-5}} = 1.434 \times 10^5$$

$$f = 0.00791\ (1.434 \times 10^5)^{0.12} = 0.03288$$

$$Sc = \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}} = \frac{1.569 \times 10^{-5}}{2.6 \times 10^{-5}} = 0.603$$

$$\therefore \frac{k_c}{V_\infty} (0.603)^{2/3} = \frac{0.03288}{2}$$

$$\therefore \frac{k_c}{V_\infty} = 0.023$$

Substitute in equation (1) we can get $c_{A_L} = 0.716\ mol/L$

Notes about this problem:

a) If the equation $f = 0.00791\ Re^{0.12}$ is not given, the friction factor can be obtained from Moody chart by using the value of the given surface roughness.

b) The diffusion coefficient may be not given. In this case you have to use the Hirschfelder equation to calculate it.

References:

1. Kothandaraman C.P "Fundamentals of Heat and Mass Transfer" New Age International, New Delhi,4th Edition 2012 (Unit I, II, III, IV, V).
2. Frank P. Incropera and David P. DeWitt, "Fundamentals of Heat and Mass Transfer", John Wiley and Sons, New Jersey,6th Edition1998(Unit I,II,III,IV, V)
3. MIT open courseware – https://ocw.mit.edu/courses/mechanical-engineering

Other web sources