Topic - Convective Mass Transfer –Flat plate

Definition of convective mass transfer:

The transport of material between a boundary surface and a moving fluid or between two immiscible moving fluids separated by a mobile interface

Convection is divided into two types:

1. Forced convection
2. Natural convection

Forced convection:

In this type the fluid moves under the influence of an external force (pressure difference) as in the case of transfer of liquids by pumps and gases by compressors.

Natural convection:

Natural convection currents develop if there is any variation in density within the fluid phase. The density variation may be due to temperature differences or to relatively large concentration differences.

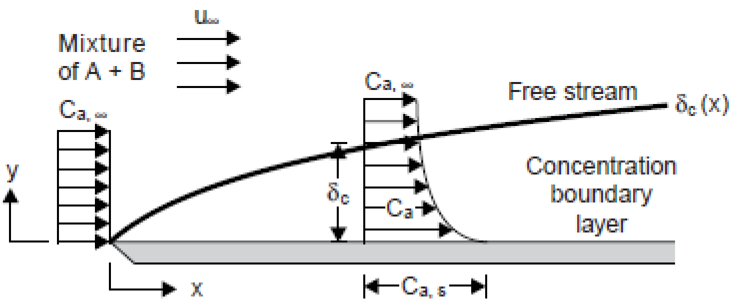
The rate equation:

The rate equation for convective mass transfer (either forced or natural) is:

NA	is the molar-mass flux of species A, measured relative to fixed spatial coordinates
kc	is the convective mass-transfer coefficient
ΔcA	is the concentration difference between the boundary surface concentration and the average concentration of the diffusing species in the moving fluid stream

When a medium deficient in a component flows over a medium having an abundance of the component, then the component will diffuse into the flowing medium. Diffusion in the opposite direction will occur if the mass concentration levels of the component are interchanged. In this case a boundary layer develops and at the interface mass transfer occurs by molecular diffusion (In heat flow at the interface, heat transfer is by conduction). Velocity boundary layer is used to determine wall friction. Thermal boundary layer is used to determine convective heat transfer. Similarly concentration boundary layer is used to determine convective mass transfer.

flow of a mixture of components A and B with a specified constant concentration over a surface rich in component A. A concentration boundary layer develops. The concentration gradient varies from the surface to the free stream. At the surface the mass transfer is by diffusion. Convective mass transfer coefficient h_m is defined by the equation, where h_m has a unit of m/s.



$$\text{Mole flow} = h_m(C_{as} - C_{a\infty})$$

The condition for diffusion at the surface is given by

$$\text{Mole flow} = -D_{ab}\left.\frac{\partial C_a}{\partial y}\right|_{y=0}$$

$$\therefore \quad h_m = \frac{-D_{ab}\cdot\left.\frac{\partial C_a}{\partial y}\right|_{y=0}}{C_{as} - C_{a\infty}} \quad \text{...(14.13)}$$

In the above case, if mass flow is to be used then

$$h_m = \frac{-D_{ab}\cdot\left.\frac{\partial \rho_a}{\partial y}\right|_{y=0}}{\rho_{as} - \rho_{a\infty}} \quad \text{...(14.14)}$$

Similar to the momentum and energy equation, the mass concentration equation can be obtained as below:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\cdot\frac{\partial^2 u}{\partial y^2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

$$u\frac{\partial C_a}{\partial x} + v\frac{\partial C_a}{\partial y} = D_{ab}\frac{\partial^2 C_a}{\partial y^2} \quad \text{...(14.15)}$$

By similarity the solutions for boundary layer thickness for connective mass transfer can be obtained. This is similar to the heat transfer by analogy. In this case, in the place of Prandtl number Schmidt number defined by

$$Sc = \nu/D_{ab} \quad \text{...(14.16)}$$

Nondimensionalising the equation leads to the condition as below:

$$\delta_m = f(Re, Sc) \quad \text{...(14.17)}$$

$$Sh = f(Re, Sc) \quad \text{...(14.18)}$$

where Sherwood number Sh is defined as

$$Sh = \frac{h_m x}{D_{ab}}.$$

In the laminar region flow over plate :

$$\delta_{mx} = \frac{5x}{Re_x^{1/2}}\cdot Sc^{-1/3} \quad \text{...(14.19)}$$

$$Sh_x = \frac{h_{mx}x}{D_{ab}} = 0.332\ Re_x^{1/2}\ Sc^{1/3} \quad \text{...(14.20)}$$

$$\overline{Sh}_L = \frac{h_m L}{D_{ab}} = 0.664\ Re_2^{1/2}\ Sc^{1/3} \quad \text{...(14.21)}$$

In the turbulent region Re > 5 × 10⁵,

$$\delta_m = \delta_v \qquad ...(14.22)$$

$$Sh_x = 0.0296\, Re_x^{0.8}\, Sc^{1/3} \qquad ...(14.23)$$

$$\overline{Sh}_L = 0.037\, Re_L^{0.8}\, Sc^{1/3} \qquad ...(14.24)$$

For flow through tubes,
In the laminar region, Re < 2000
For uniform wall mass concentration,

$$Sh = 3.66 \qquad ...(14.25)$$

For uniform wall mass flux

$$Sh = 4.36 \qquad ...(14.26)$$

For turbulent region,

$$Sh = 0.023\, Re^{0.83}\, Sc^{1/3} \qquad ...(14.27)$$

Problem 1:

In applying dimensional analysis to explain mass-transfer coefficient, one must consider the geometry involved, a variable to explain the flow characteristics of the moving stream, and the properties of the moving stream. Predict the variables that are necessary to explain the mass-transfer coefficient for a gas stream flowing over a flat plate and arrange these variables into dimensionless groups.

Solution:

To get the dimensionless groups that will be used to predict the mass transfer coefficient, we will follow the steps of dimensional analysis:

1. **Identify the important variables in the system**

The important variables are: $L, V, \mu, \rho, D_{AB}, k_c$

2. **List all the problem variables and parameters, along with their dimensions**

Quantity	Dimensional expression
L	L
V	L/t
μ	M/Lt
ρ	M/L^3
D_{AB}	L^2/t
k_c	L/t

The number of dimensionless groups = number of variables – number of fundamental dimensions

The number of dimensionless groups = 6 – 3 = 3

3. **Choose a set of reference variables**

Choose a set of reference variables. The choice of variables is arbitrary, except that the following criteria must be satisfied:

- The number of reference variables must be equal to the minimum number of fundamental dimensions in the problem (in this case, three).

- No two reference variables should have exactly the same dimensions.
- All the dimensions that appear in the problem variables must also appear somewhere in the dimensions of the reference variables.

__Note:__ In selection of the reference variable, try to select the variable of the simplest dimensions.

In this case the reference variables will be: L, V and ρ

$$L = L$$

$$V = L/t$$

$$\rho = M/L^3$$

4. **Solve the dimensional equations for the dimensions (L, m and t) in terms of the reference variables**

$$L = L$$

$$t = d/V$$

$$M = \rho d^3$$

6. **Write the dimensional equations for each of the remaining variables in terms of the reference variables**

$\mu = \frac{M}{Lt} = \frac{\rho d^3}{L(L/V)} = \rho VL$	(1)
$D_{AB} = \frac{L^2}{t} = \frac{L^2}{L/V} = VL$	(2)
$k_c = \frac{L}{t} = \frac{L}{L/V} = V$	(3)

Divide equation (1) by equation (2) and equation (3) by equation (2)

$\frac{\mu}{D_{AB}} = \rho$	(4)
$\frac{k_c}{D_{AB}} = \frac{1}{L}$	(5)

7. **The resulting equations are each a dimensional identity, so dividing one side by the other results in one dimensionless group from each equation**

From equations (1), (4) and (5)

$$N_1 = \frac{\rho V d}{\mu} = Re$$

$$N_2 = \frac{\mu}{\rho D_{AB}} = Sc$$

$$N_3 = \frac{k_c d}{D_{AB}} = Sh$$

References:

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3. MIT open courseware – https://ocw.mit.edu/courses/mechanical-engineering

Other web sources