UNIT – III

Two Dimensional Problems

Scalar variable problems

Plane Stresses :-

Plane stress is defined to be a state of stress in which normal stress (σ) and shear stress (τ) directed perpendicular to the plane (or) assumed to be zero.

Plate with holes and Plates with Fillet are comming under Plain stress analysis problem.

$$\sigma_z = 0$$

$$\tau_{xz} \ \& \ \tau_{yz} = 0$$

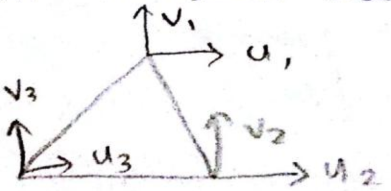
Plane strain Analysis :-

Plane strain is defined to be a state of strain in which the strain normal to the xy plane and shear strain are assumed to be zero.

$$e_z = 0$$

$$\gamma_{xz} \ \& \ \gamma_{yz} = 0.$$

Constant Strain triangular Element (CST).



Two Dimensional Problems

Formulae used

1. For constant strain triangle (CST) element

Shape function, $N_1 + N_2 + N_3 = 1$

Co-ordinate, $x = N_1 x_1 + N_2 x_2 + N_3 x_3$

Co-ordinate, $y = N_1 y_1 + N_2 y_2 + N_3 y_3$

(or)

Co-ordinate, $x = (x_1 - x_3) N_1 + (x_2 - x_3) N_2 + x_3$

Co-ordinate, $y = (y_1 - y_3) N_1 + (y_2 - y_3) N_2 + y_3$

2. Area of the triangular element, $A = \frac{1}{2} \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$

3. Strain-displacement matrix for CST element is,

$$[B] = \frac{1}{2A} \begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix}$$

Where, $q_1 = y_2 - y_3$; $q_2 = y_3 - y_1$; $q_3 = y_1 - y_2$

$r_1 = x_3 - x_2$; $r_2 = x_1 - x_3$; $r_3 = x_2 - x_1$

4. Stress-strain relationship matrix for plane stress problem,

$$[D] = \frac{E}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

Where, ν = Poison's ratio

$E \rightarrow$ young's modulus.

5. Stress- Strain relationship matrix for plane strain problem,

$$[D] = \frac{E}{(1+v)(1-2v)} \begin{bmatrix} 1-v & v & 0 \\ v & 1-v & 0 \\ 0 & 0 & \frac{1-2v}{2} \end{bmatrix}$$

6. Element stiffness matrix for CST Element,

$$[K] = [B]^T [D] [B] A t$$

7. Element stress, $\{\sigma\} = [D] [B] \{u\}$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [D] [B] \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$

$\sigma_x, \sigma_y \rightarrow$ normal stresses

$\tau_{xy} \rightarrow$ shear stress,

$u, v \rightarrow$ Nodal displacements

8. Maximum normal stress, $\sigma_{max} = \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Minimum normal stress, $\sigma_{min} = \sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

9. Principle angle $\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

10. Element strain, $\{e\} = [B] \{u\} = [B] \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$

11. Temperature effects,

Initial strain, $\{e_0\}$ (For plane stress problems) $= \begin{Bmatrix} \alpha \Delta T \\ \alpha \Delta T \\ 0 \end{Bmatrix}$

or

(For plane strain problems) $= (1+v) \begin{Bmatrix} \alpha \Delta T \\ \alpha \Delta T \\ 0 \end{Bmatrix}$

$\alpha \rightarrow$ co-efficient of thermal expansion

$v \rightarrow$ Poisson's ratio.

12. Element temperature force, $\{F\} = [B]^T [D] \{e_0\} t A$,