



#### Problem :

- ① An experiment was designed to study the performance of 4 different detergents for cleaning of injectors. The following "cleanliness" readings were obtained with specially designed equipment for 12 tanks of gas distributed over 3 different models of engines.

Detergent	Engine 1	Engine 2	Engine 3	Total
A	45	43	51	139
B	47	46	52	145
C	48	50	55	153
D	42	37	49	128
Total	182	176	207	565

Perform the ANOVA test at 0.01 level of significance whether there are differences in the detergents or in the engines.

Solution:

Fix origin = 50. Subtract each element from 50.

Engine Detergent	$x_1$	$x_2$	$x_3$	Total	$x_1^2$	$x_2^2$	$x_3^2$
A ( $y_1$ )	-5	-7	+1	-11	25	49	1
B ( $y_2$ )	-3	-4	2	-5	9	16	4
C ( $y_3$ )	-2	0	5	3	4	0	25
D ( $y_4$ )	-8	-13	-1	-22	64	169	1
Total	-18	-24	7	-35	102	234	31



Step 1 : Null hypothesis  $H_0$  : There is no Significant difference between engines and detergents.

Alternative hypothesis  $H_1$  : There is a significant difference between engines and detergents

Step 2 : \*  $N = 12$

$$T = -35$$

$$C.F = T^2/N = 102.08$$

Step 3 :  $SST = \sum x_1^2 + \sum x_2^2 + \sum x_3^2 - C.F$

$$= 102 + 234 + 31 - 102.08$$

$$\boxed{SST = 264.92}$$

$$SSC = \frac{(\sum x_1)^2}{C_1} + \frac{(\sum x_2)^2}{C_2} + \frac{(\sum x_3)^2}{C_3} - C.F$$

$$= \frac{(-18)^2}{4} + \frac{(-24)^2}{4} + \frac{7^2}{4} - 102.08$$

$$\boxed{SSC = 135.17}$$

$$SSR = \frac{(\sum y_1)^2}{r_1} + \frac{(\sum y_2)^2}{r_2} + \frac{(\sum y_3)^2}{r_3} + \frac{(\sum y_4)^2}{r_4} - C.F$$

$$= \frac{(-11)^2}{3} + \frac{(-5)^2}{3} + \frac{3^2}{3} + \frac{(-22)^2}{3} - 102.08$$

$$\boxed{SSR = 110.91}$$

$$SSE = TSS - SSC - SSR = 264.92 - 135.17 - 110.91$$

$$\boxed{SSE = 18.84}$$

Step 4 : ANOVA table :

Source of Variation	Degree of freedom	Sum of Squares	Mean Sum of Squares	Variance Ratio	Table Value at 1% level
Between Columns	$C-1 = 3-1 = 2$	$SSC = 135.17$	$MSC = \frac{SSC}{C-1} = 67.585$	$F_c = \frac{MSC}{MSE} = 31.52$	$F_{\alpha}(2,6) = 10.92$
Between rows	$r-1 = 4-1 = 3$	$SSR = 110.91$	$MSR = \frac{SSR}{r-1} = 36.97$	$F_r = \frac{MSR}{MSE} = 11.77$	$F_{\alpha}(3,6) = 9.78$
Between errors	$(C-1)(r-1) = 6$	$SSE = 18.84$	$MSE = \frac{SSE}{(C-1)(r-1)} = 3.14$		

Steps : Decision : Since  $F_c > F_{\alpha}$  and  $F_r > F_{\alpha}$ ,  $H_0$  is rejected

$\therefore$  There is a significant difference between engines and detergents