LATIN SQUARE :

1) An agriculturist wants to test the effects of four different fertilizers A, B, c and D on the yield of paddy. In order to eliminate sources of error due to variability in soil-fertility he used the fertilizers in a Latin square arrangements given below where the numbers indicate yields in quintals per unit area. Perform an analysis of variance to decide whether there is a difference between the fertilizers at 5% level of significance.

A18	D21	C23	B11
D22	A20	B10	C19
B15	C21	D25	A17
C22	B12	A15	D24

Soln: Let Origin = $n_{ij} - 18$. avg(min, max)

	x_1	x_2	x_3	x_4	Total	n_1^2	n_2^2	n_3^2	n_4^2
y_1	0	3	5	-7	1	0	9	25	49
y_2	4	+2	-8	1	-1	16	4	64	1
y_3	-3	3	7	-1	6	9	9	49	1
y_4	4	-6	-3	6	1	16	36	9	36
Total	5	2	1	-1	7	41	58	147	87
	Σn_1	Σn_2	Σn_3	Σn_4		Σn_1^2	Σn_2^2	Σn_3^2	Σn_4^2

Step1: Formulate H_0 & H_1:

H_0: There is no difference between the fertilizers

H_1: There is difference between the fertilizers.

step 2 : To find T & N :

$$T = \Sigma x_1 + \Sigma n_2 + \Sigma n_3 + \Sigma n_4.$$

$$= 5 + 2 + 1 - 1 = 7$$

$$N = n_1 + n_2 + n_3 + n_4$$

$$= 4 + 4 + 4 + 4 = 16$$

step 3 : To find correction factor, C.F

$$C.F = \frac{T^2}{N} = \frac{49}{16} = 3.0625$$

step 4 : To find TSS :

$$TSS = \Sigma n_1^2 + \Sigma n_2^2 + \Sigma n_3^2 + \Sigma n_4^2 - CF$$

$$= 41 + 58 + 147 + 87 - 3.0625$$

$$= 333 - 3.0625 \qquad = 329.94$$

step 5 : To find SSC, SSR, & SST

$$SSC = \frac{(\Sigma n_1)^2}{n_1} + \frac{(\Sigma n_2)^2}{n_2} + \frac{(\Sigma n_3)^2}{n_3} + \frac{(\Sigma x_4)^2}{n_4} - C.F$$

$$= \frac{5^2}{4} + \frac{2^2}{4} + \frac{1^2}{4} + \frac{-1^2}{4} - 3.0625$$

$$= 4.6875$$

$$SSR = \frac{(\Sigma y_1)^2}{n_1'} + \frac{(\Sigma y_2)^2}{n_2'} + \frac{(\Sigma y_3)^2}{n_3'} + \frac{(\Sigma y_4)^2}{n_4'} - C.F$$

$$= \frac{1^2}{4} + \frac{-1^2}{4} + \frac{6^2}{4} + \frac{1^2}{4} - 3.0625$$

$$= 6.6875$$

To find SST :

A	0	2	-3	-1	$-2 : \Sigma z_1$
B	-3	-6	-8	-7	$-24 : \Sigma z_2$
C	4	3	5	1	$13 : \Sigma z_3$
D	4	3	7	6	$20 \quad \Sigma z_4$

$$SST = \frac{(\Sigma z_1)^2}{4} + \frac{(\Sigma z_2)^2}{4} + \frac{(\Sigma z_3)^2}{4} + \frac{(\Sigma z_4)^2}{4} - c.f.$$

$$= \frac{-2^2}{4} + \frac{-24^2}{4} + \frac{13^2}{4} + \frac{20^2}{4} - c.f.$$

$$= 287.25 - 3.0625 = 284.1875$$

step 6: To find SSE

$$SSE = TSS - SSC - SSR - SST$$

$$= 329.94 - 4.6875 - 6.6875 - 284.1875$$

$$= 34.375$$

Step 7: Annova table.

Source of variations	Sum of squares	Degrees of Freedom	Mean Sum of squares	F-Ratio
Column	SSC : 4.6875	$c-1 = 3$	$MSC = \dfrac{4.6875}{3} = 1.5625$	$F_C : \dfrac{5.729}{1.56}$ $= 3.6666$ $F_\alpha(6,3) = 9$
Row	SSR : 6.6875	$r-1 = 3$	$MSR : \dfrac{6.6875}{3} = 2.2291$	$F_R : \dfrac{5.729}{0.229}$ $= 2.5701$ $F_\alpha(6,5) = 9$
Treatment	SST : 284.1875	$T-1 = 3$	$MST : \dfrac{284.1875}{3} = 94.7291$	$F_T : \dfrac{94.7}{5.729}$ $= 16.53$ $F_\alpha(3,6) :$ 4.7
Error	SSE : 34.375	$(n-1)(n-2)$ $3 \times 2 = 6$	$MSE : \dfrac{34.375}{6} = 5.7291$	

Step 8: Conclusion:

$F_C = 3.6666 < 9.94 = F_\alpha$, Ho is accepted

$F_R = 2.5701 < 9.94 = F_\alpha$, Ho is accepted

$F_T = 16.5347 > 4.76 = F_\alpha$, Ho is rejected

(or) There is difference between the fertilizers.

2) Analyse the variance in the Latin square of yields (in quintals) of wheat where P, Q, R, S represent the different manures used.

S 222	P 221	R 223	Q 222
Q 224	R 223	P 222	S 225
P 220	Q 219	S 220	R 221
R 222	S 223	Q 221	P 222

Test whether the different manures used have given significantly different yields.

Soln: $F_C : 1.37$; $F_R : 12.31$, $F_T : 2.12$ & $F_\alpha : 4.76$.