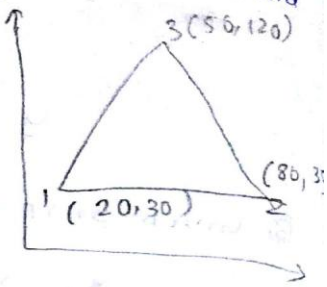
(8) Determine the stiffness matrix for the CST element the co-ordinates are given in units of mm. Assume plane stress conditions. Take E = 210 GPa Poisson's ratio $\nu = 0.25$, thickness $t = 10$ mm.



To Find :

Stiffness matrix K.

Sol :-

$$[k] = [B]^T [D] [B] A \times t$$

where,

B = Strain displacement matrix

D = Stress strain relationship matrix

A = Area

t = thickness

$$\text{Area } (A) = \frac{1}{2} \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

$x_1 = 20, \; y_1 = 30$

$x_2 = 80, \; y_2 = 30$

$x_3 = 50, \; y_3 = 120$

$$A = \frac{1}{2} \begin{vmatrix} 1 & 20 & 30 \\ 1 & 80 & 30 \\ 1 & 50 & 120 \end{vmatrix}$$

$$= \frac{1}{2} \left[1 (9600 - 1500) - 20 (120 - 30) + 30 (50 \right.$$

$$= \frac{1}{2} \left[8100 - 1800 - 900 \right]$$

$$A = 2700 \text{ mm}^2$$

Strain displacement matrix [B]

$$[B] = \frac{1}{2A}\begin{vmatrix} Q_1 & 0 & Q_2 & 0 & Q_3 & 0 \\ 0 & R_1 & 0 & r_2 & 0 & r_3 \\ r_1 & Q_1 & r_2 & Q_2 & r_3 & Q_3 \end{vmatrix}$$

$q = 2, 3, 1 \quad , \quad r = 3, 2, 1$

$q_1 = (y_2 - y_3) = (30 - 120) = -90$

$q_2 = (y_3 - y_1) = (120 - 30) = 90$

$q_3 = (y_1 - y_2) = (30 - 30) = 0$

$r_1 = x_3 - x_2 = (50 - 80) = -30$

$r_2 = x_1 - x_3 = (20 - 50) = -30$

$r_3 = x_2 - x_1 = (80 - 20) = 60$

$$[B] = \frac{1}{2A}\begin{vmatrix} -90 & 0 & 90 & 0 & 0 & 0 \\ 0 & -30 & 0 & -30 & 0 & 60 \\ -30 & -90 & -30 & 90 & 60 & 0 \end{vmatrix}$$

$$= \frac{1}{2\times2700}\begin{vmatrix} -90 & 0 & 90 & 0 & 0 & 0 \\ 0 & -30 & 0 & -30 & 0 & 60 \\ -30 & -90 & -30 & 90 & 60 & 0 \end{vmatrix}$$

$$= 5.55\times10^{-3}\begin{vmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{vmatrix}$$

$$[D] = \frac{E}{1-\nu^2}\begin{vmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{vmatrix}$$

$$[D] = \frac{21 \times 10^5}{1 - 0.25^2} \begin{vmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & \frac{1-0.25}{2} \end{vmatrix}$$

$$= 56 \times 10^3 \begin{vmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{vmatrix}$$

(B) $[D][B] = 56 \times 10^3 \begin{vmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{vmatrix} \times 5.55 \times 10^{-3} \begin{vmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{vmatrix}$

$= 310.8 \begin{vmatrix} -12+0+0 & 0-1+0 & 12+0+0 & 0-1+0 & 0+0+0 & 0+2+0 \\ -3+0+0 & 0-4+0 & 3+0+0 & 0-4+0 & 0+0+0 & 0+8+0 \\ 0+0-1.5 & 0+0-4.5 & 0+0-1.5 & 0+0-4.5 & 0+0+3 & 0+16+0 \end{vmatrix}$

$= 310.8 \begin{vmatrix} -12 & -1 & 12 & -1 & 0 & 2 \\ -3 & -4 & 3 & -4 & 0 & 8 \\ -1.5 & -4.5 & -1.5 & -4.5 & 3 & 0 \end{vmatrix}$

(B) $(B)^T [D][B] = 5.55 \times 10^{-3} \begin{bmatrix} -3 & 0 & -1 \\ 0 & -1 & -3 \\ 3 & 0 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} \times 310.8 \begin{bmatrix} -12 & -1 & 12 & -1 & 0 & 2 \\ -3 & -4 & 3 & -4 & 0 & 8 \\ -1.5 & -4.5 & -1.5 & -4.5 & 3 & 0 \end{bmatrix}$

$$[B]^T [D] [B] = 1.72494 \begin{bmatrix} 37.5 & 7.5 & -34.5 & -1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -34.5 & 1.5 & 37.5 & -7.5 & -3 & 6 \\ -1.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -3 & -9 & -3 & 9 & 6 & 0 \\ -6 & -8 & 6 & -8 & 0 & 16 \end{bmatrix}$$

$$K = [B]^T [D] [B] A \times t$$

$$= 1.72494 \times 2700 \times 10 \begin{bmatrix} u \end{bmatrix}$$

$$= 46.573 \times 10^3 \begin{bmatrix} u \end{bmatrix}$$