UNIT- IV

Heat Transfer

Heat transfer can be defined as the transmission of energy from one region to another region due to temperature difference. (The temperature distribution within a body).

There are Three modes of heat transfer,

(i) conduction,

(ii) convection

(iii) Radiation.

Conduction :

Heat conduction is a mechanism of heat transfer from a region of high temperature to a region of low temperature within a medium (solid, liquid or gases).

Convection :

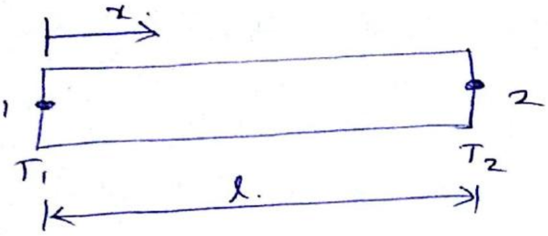
convection is a process of heat transfer that will occur between a solid surface and a fluid medium when they are at different temperatures.

Radiation :

The heat transfer from one body to another without any transmitting medium.

(It is an electromagnetic wave phenomenon.

Derivation of Temperature Function (T) and shape function (N) for one dimensional Heat conduction Element :-



Consider bar element with nodes 1 and 2. T_1, T_2 are the temperature at the respective nodes. So, T_1 and T_2 are considered as DOF of this bar element.

Since the element has got two degrees of freedom, it will have two generalized co-ordinates.

$$T = a_0 + a_1 x$$

$a_0, a_1 \rightarrow$ Global (or) generalized co-ordinates.

$$T = \begin{bmatrix} 1 & x \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} \longrightarrow (1)$$

At node 1 ; $T = T_1$, $x = 0$

At node 2 : $T = T_2$, $x = l$.

$$T_1 = a_0,$$

$$T_2 = a_0 + a_1 l.$$

Write down the above equations in the Matrix form.

$$\begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & l \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix}$$

$$\begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & l \end{bmatrix}^{-1} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix}$$

$$\left\{ \text{Note: } \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{1}{(a_{11} a_{22} - a_{12} a_{21})} \times \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \right\}$$

$$\begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = \frac{1}{l} \begin{bmatrix} l & 0 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix}$$

substitute $\begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix}$ values in equation (1)

$$T = [1 \;\; x] \frac{1}{l} \begin{bmatrix} l & 0 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix}$$

$$= \frac{1}{l} [1 \;\; x] \begin{bmatrix} l & 0 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix}$$

$$= \frac{1}{l} [l - x \;\;\; 0 + x] \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix}$$

$$= \begin{bmatrix} \frac{l - x}{l} & \frac{x}{l} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix}$$

$$T = [N_1 \;\; N_2] \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix}$$

Temperature function $T = N_1 T_1 + N_2 T_2$

where, shape functions, $N_1 = \frac{l - x}{l}$,

$N_2 = x / l$