Axisymmetric Elements

The special two dimensional element called axisymmetric element

Many three dimensional problems in engineering exhibit symmetry about an axis of rotation. Such types of problems are known as axisymmetric problems.

These problems can be solved by using two dimensional finite elements.

These elements are most conveniently described in cylinderical (r, θ, z) co-ordinates.

The required conditions for a problems to be axisymmetric.

1. The problem domain must be symmetric about the axis of revolution, which is conventionally taken as the z-axis
2. All the boundry conditions must be symmetric about the axis of revolution.
3. All loading conditions must be symmetric about the axis of revolution.

$$\text{Stress, } \{\sigma\} = \begin{Bmatrix} \sigma_r \rightarrow \text{radial} \\ \sigma_\theta \rightarrow \text{longitudinal} \\ \sigma_z \rightarrow \text{circumferential} \\ \tau_{rz} \rightarrow \text{shear stress} \end{Bmatrix} \qquad \{e\} = \begin{Bmatrix} e_r \\ e_\theta \\ e_z \\ \gamma_{rz} \end{Bmatrix}$$

$$\text{Force} = \begin{Bmatrix} F_r \\ F_z \end{Bmatrix}$$

$$\text{Area } A = \frac{1}{2} \begin{vmatrix} 1 & r_1 & z_1 \\ 1 & r_2 & z_2 \\ 1 & r_3 & z_3 \end{vmatrix}$$

Strain - Displacement matrix

$$[B] = \frac{1}{2A} \begin{bmatrix} \beta_1 & 0 & \beta_2 & 0 & \beta_3 & 0 \\ \frac{\alpha_1}{r}+\beta_1+\frac{\gamma_1 z}{r} & 0 & \frac{\alpha_2}{r}+\beta_2+\frac{\gamma_2 z}{r} & 0 & \frac{\alpha_3}{r}+\beta_3+\frac{\gamma_3 z}{r} & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\ \gamma_1 & \beta_1 & \gamma_2 & \beta_2 & \gamma_3 & \beta_3 \end{bmatrix}$$

$$r = \frac{r_1+r_2+r_3}{3}, \quad z = \frac{z_1+z_2+z_3}{3}$$

$$\alpha_1 = r_2 z_3 - r_3 z_2 \qquad \beta_1 = z_2 - z_3 \qquad \gamma_1 = r_3 - r_2$$

$$\alpha_2 = r_3 z_1 - r_1 z_3 \qquad \beta_2 = z_3 - z_1 \qquad \gamma_2 = r_1 - r_3$$

$$\alpha_3 = r_1 z_2 - r_2 z_1 \qquad \beta_3 = z_1 - z_2 \qquad \gamma_3 = r_2 - r_1$$

$$[K] = 2\pi r A \, [B]^T [D] [B].$$

Stress - strain relationship matrix [D]

$$[D] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$