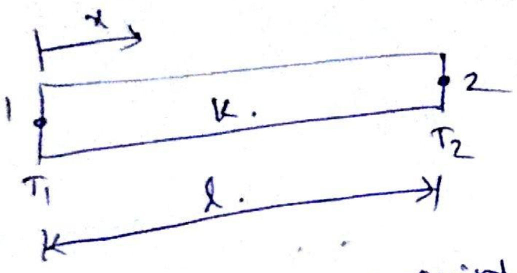
Derivation of stiffness matrix for One-Dimensional Heat Conduction Element:



Where, $k \rightarrow$ Thermal conductivity of the materials,

$T_1, T_2 \rightarrow$ Temperatures at the respective nodes.

WKT,

Stiffness matrix $[k] = \int_V [B]^T \cdot [D] [B] \, dv$.

In one dimensional element,

Temperature function, $T = N_1 T_1 + N_2 T_2$.

Where, $N_1 = \frac{l - x}{l}$.

$N_2 = x/l$.

WKT,

Strain displacement matrix, $[B] = \begin{bmatrix} \frac{dN_1}{dx} & \frac{dN_2}{dx} \end{bmatrix}$

$[B] = \begin{bmatrix} \frac{-1}{l} & \frac{1}{l} \end{bmatrix}$

$[B]^T = \begin{Bmatrix} -1/l \\ 1/l \end{Bmatrix}$

In one dimensional heat conduction problems, ③.

Substitute [B], $[B]^T$ and [D] values in stiffness matrix equation,

$$[K_c] = \int_0^l \begin{Bmatrix} -1/l \\ 1/l \end{Bmatrix} \times K \times \begin{bmatrix} \frac{-1}{l} & \frac{1}{l} \end{bmatrix} dv$$

$$= \int_0^l \begin{bmatrix} \frac{1}{l^2} & -\frac{1}{l^2} \\ -1/l^2 & 1/l^2 \end{bmatrix} K \, dv \qquad \{dv = A \times dx\}$$

$$= \int_0^l \begin{bmatrix} 1/l^2 & -1/l^2 \\ -1/l^2 & 1/l^2 \end{bmatrix} K.A.dx.$$

$$= A K \int_0^l \begin{bmatrix} 1/l^2 & -1/l^2 \\ -1/l^2 & 1/l^2 \end{bmatrix} dx.$$

$$= A.K \begin{bmatrix} 1/l^2 & -1/l^2 \\ -1/l^2 & 1/l^2 \end{bmatrix} . [x]_0^l$$

$$= A K \begin{bmatrix} 1/l^2 & -1/l^2 \\ -1/l^2 & 1/l^2 \end{bmatrix} (l-0).$$

$$[K_c] = \frac{AK}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

where,

A = Area of the element, m^2