



The nodal co ordinates for an axis symmetric triangular element are given by  $r_1 = 10 \text{ mm}$ ,  $z_1 = 10 \text{ mm}$ ,  $r_2 = 30 \text{ mm}$ ,  $z_2 = 10 \text{ mm}$ ,  $r_3 = 30 \text{ mm}$ ,  $z_3 = 40 \text{ mm}$

$$A = \frac{1}{2} \begin{vmatrix} 1 & r_1 & z_1 \\ 0 & 1 & r_2 & z_2 \\ 0 & 1 & r_3 & z_3 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} 1 & 10 & 10 \\ 0 & 1 & 30 & 10 \\ 0 & 1 & 30 & 40 \end{vmatrix} = \frac{1}{2} \left[ 1(1200 - 300) - 10(40 - 10) + 10(30 - 30) \right]$$

$$= 300 \text{ mm}^2$$

$$\text{co ordinates } r = \frac{r_1 + r_2 + r_3}{3} = \frac{10 + 30 + 30}{3} = 23.3 \text{ mm}$$

$$z = \frac{z_1 + z_2 + z_3}{3} = \frac{10 + 10 + 40}{3} = 20 \text{ mm}$$

$$\alpha_1 = r_2 z_3 - r_3 z_2 = (1200 - 300) = 900 \text{ mm}^2$$

$$\alpha_2 = r_3 z_1 - r_1 z_3 = -100 \text{ mm}^2$$

$$\alpha_3 = r_1 z_2 - r_2 z_1 = -200 \text{ mm}^2$$

$$\beta_1 = z_2 - z_3 = -30 \text{ mm}$$

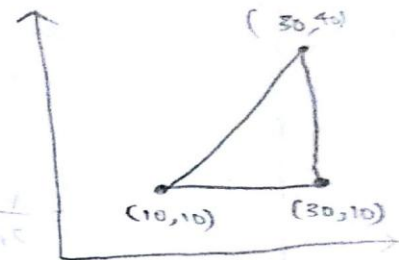
$$\beta_2 = z_3 - z_1 = 30 \text{ mm}$$

$$\beta_3 = z_1 - z_2 = 0 \text{ mm}$$

$$\gamma_1 = r_3 - r_2 = 0 \text{ mm}$$

$$\gamma_2 = r_1 - r_3 = -30 \text{ mm}$$

$$\gamma_3 = r_2 - r_1 = 20 \text{ mm}$$





Strain displacement Matrix,

$$[B] = \frac{1}{2A}$$

$$\begin{bmatrix} P_1 & 0 & P_2 & 0 & P_3 & 0 \\ \frac{\alpha_1}{\gamma} + P_1 + \frac{\gamma_1^2}{\gamma} & 0 & \frac{\alpha_2}{\gamma} + P_2 + \frac{\gamma_2^2}{\gamma} & 0 & \frac{\alpha_3}{\gamma} + P_3 + \frac{\gamma_3^2}{\gamma} & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\ \gamma_1 & P_1 & \gamma_2 & P_2 & \gamma_3 & P_3 \end{bmatrix}$$

$$= \frac{1}{2 \times 300} \begin{bmatrix} -30 & 0 & 30 & 0 & 0 & 0 \\ 8.576 & 0 & 18.568 & 0 & -8.572 & 0 \\ 0 & 0 & 0 & -20 & 0 & 0 \\ 0 & -30 & -20 & 30 & 0 & 0 \end{bmatrix}$$

$$[B] = \begin{bmatrix} -0.05 & 0 & 0.05 & 0 & 0 & 0 \\ 0.0142 & 0 & 0.0142 & 0 & -0.0142 & 0 \\ 0 & 0 & 0 & -0.033 & 0 & 0 \\ 0 & -0.05 & -0.033 & 0.05 & 0 & 0 \end{bmatrix}$$