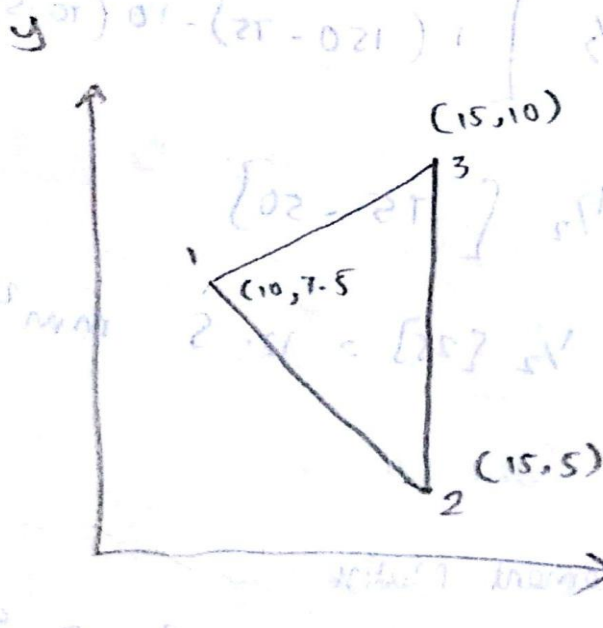




Calculate the element stresses σ_x , σ_y , τ_{xy} and the principle angle θ_p for the element as shown.



The nodal displacements are $u_1 = 2 \text{ mm}$, $v_1 = 1 \text{ mm}$, $u_2 = 0.5 \text{ mm}$, $v_2 = 0 \text{ mm}$, $u_3 = 3 \text{ mm}$, $v_3 = 1 \text{ mm}$.
take $E = 2.1 \times 10^5 \text{ N/mm}^2$ and $\nu = 0.25$. assume



Plane stress condition.

To find:

σ_x - normal stress

σ_y - normal stress

τ_{xy} - shear stress

σ_1 - Maximum normal stress

σ_2 - minimum "

θ_p - Principle angle.

Sol:

$$\text{Area} = \frac{1}{2} \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} 1 & 10 & 7.5 \\ 1 & 15 & 5 \\ 1 & 15 & 10 \end{vmatrix}$$

$$= \frac{1}{2} [1(150 - 75) - 10(10 - 5) + 7.5(15 - 15)]$$

$$= \frac{1}{2} [75 - 50]$$

$$\text{Area} = \frac{1}{2} [25] = 12.5 \text{ mm}^2$$

Stress displacement matrix

$$[B] = \frac{1}{2h} \begin{bmatrix} x_1 & 0 & x_2 & 0 & x_3 & 0 \\ 0 & y_1 & 0 & y_2 & 0 & y_3 \\ x_1 & y_1 & x_2 & y_2 & x_3 & y_3 \end{bmatrix}$$

$$q_1 = (y_2 - y_3) = (5 - 10) = -5$$



$$q_2 = (y_3 - y_1) = (10 - 7.5) = 2.5$$

$$q_3 = (y_1 - y_2) = (7.5 - 5) = 2.5$$

$$r_1 = (x_3 - x_2) = (15 - 15) = 0$$

$$r_2 = (x_1 - x_3) = (10 - 15) = -5$$

$$r_3 = (x_2 - x_1) = (15 - 10) = +5$$

$$[B] = \frac{1}{2 \times 12.5} \begin{bmatrix} -5 & 0 & 2.5 & 0 & 2.5 & 0 \\ 0 & 0 & 0 & -5 & 0 & +5 \\ 0 & -5 & -5 & 2.5 & +5 & 2.5 \end{bmatrix}$$

$$= 0.04 \times \begin{bmatrix} -2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & 0 & +2 \\ 0 & -2 & -2 & 1 & 2 & 1 \end{bmatrix}$$

$$[D] = \frac{E}{\pi \sqrt{2}} \begin{bmatrix} 1 & \sqrt{2} & 0 \\ \sqrt{2} & 1 & 0 \\ 0 & 0 & \frac{1-\sqrt{2}}{2} \end{bmatrix}$$

$$= \frac{2.1 \times 10^5}{1 - (0.25)^2} \begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & 1 - \frac{0.25}{2} \end{bmatrix}$$

$$= 56 \times 10^3 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$



$$[D] \{B\} = 5.6 \times 10^3 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \times \begin{bmatrix} -2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & 0 & 2 \\ 0 & -2 & -2 & 1 & 2 & 1 \end{bmatrix}$$

$$= 5.6 \times 10^3 \begin{bmatrix} -2+0+0 & 0+0+0 & 4+0+0 & 0-2+0 & 4+0+0 & 0+0+0 \\ -2+0+0 & 0+0+0 & 1+0+0 & 0-2+0 & 1+0+0 & 0+0+0 \\ 0+0+0 & 0+0-3 & 0+0-3 & 0+0+1.5 & 0+0+3 & 0+0+1.5 \end{bmatrix}$$

$$= 5.6 \times 10^3 \begin{bmatrix} -2 & 0 & 4 & -2 & 4 & 2 \\ -2 & 0 & 1 & -2 & 1 & 8 \\ 0 & -3 & -3 & 1.5 & 3 & 1.5 \end{bmatrix}$$

$$\text{Stress } \sigma = [B] \times [D] \times \{u\}$$

$$= 5.6 \times 10^3 \begin{bmatrix} -2 & 0 & 4 & -2 & 4 & 2 \\ -2 & 0 & 1 & -2 & 1 & 8 \\ 0 & -3 & -3 & 1.5 & 3 & 1.5 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.3 \end{Bmatrix}$$

$$= 5.6 \times 10^3 \begin{bmatrix} -16+0+0+2+0+12+2 \\ -4+0+0+0+0+8 \\ 0-3-3+1.5+3+1.5 \end{bmatrix}$$

$$\sigma = 5.6 \times 10^3 \begin{bmatrix} 0 \\ 7.5 \\ 6 \end{bmatrix}$$

$$\sigma = \begin{bmatrix} 0 \\ 42000 \\ 33600 \end{bmatrix} = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

$$\sigma_x = 0$$

$$\sigma_y = 4.2 \times 10^4 \text{ N/m}^2$$

$$\tau_{xy} = 3.36 \times 10^4 \text{ N/m}^2$$

$$\sigma_{\max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

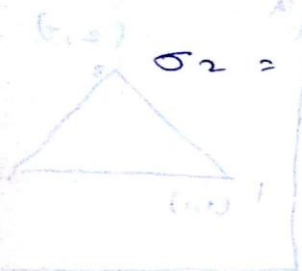


$$\sigma_{max} = \frac{0 + 4.2 \times 10^3}{2} + \sqrt{\left(\frac{0 - 4.2 \times 10^3}{2}\right)^2 + (33.6 \times 10^3)^2}$$

$$= 2.1 \times 10^3 + 39622.72$$

$$\sigma_{max} = 60.622 \times 10^3 \text{ N/mm}^2$$

minimum nominal stress, (σ_{min})



$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 2.1 \times 10^3 - 39622.72$$

$$= -18.622 \times 10^3 \text{ N/mm}^2$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta_p = \frac{2 \times 33.6 \times 10^3}{0 - 4.2 \times 10^3}$$

$$\tan 2\theta_p = -1.6$$

$$2\theta_p = \tan^{-1}(-1.6)$$

$$2\theta_p = -57.99$$

$$\theta_p = -29^\circ$$

Note:
for calculating strain, $\epsilon = [B] \times \{u\}$