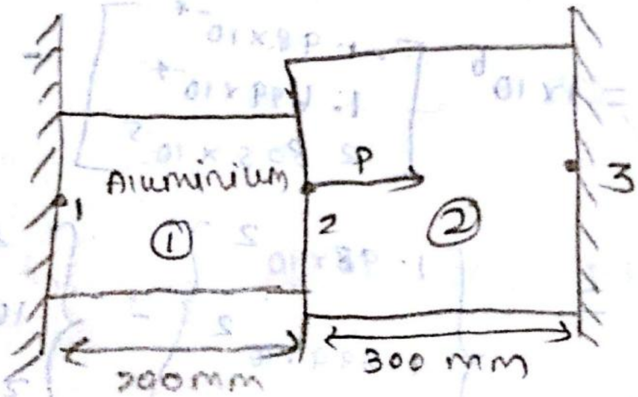
① An axial load of 4×10^5 N is applied at 30°c to the rod as shown in fig. The temperature is then raised to 60°c. calculate the following:

(i) Assemble the k and F matrices,

(ii) Nodal displacements,

(iii) Stresses in each element,

(iv) Reactions at each nodal point.



For aluminium

$A_1 = 1000 mm^2$

$E_1 = 0.7\times10^5 N/mm^2$

$\alpha_1 = 23\times10^{-6}/°c$

For steel

$A_2 = 1500 mm^2$

$E_2 = 2\times10^5 N/mm^2$

$\alpha_2 = 12\times10^{-6}/°C.$

Given data :

$P = 4\times10^5$ N , $\Delta T = 60-30 = 30^\circ C$

$l_1 = 200$ mm, $l_2 = 300$ mm.

To Find :

(i) Assemble the [K] and {F} matrices

(ii) u_1, u_2, u_3

(iii) σ_1, σ_2

(iv) R_1, R_2, R_3.

Sol :

Finite element equation

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

For element 1 : Nodes (1, 2)

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{A_1 E_1}{l_1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= \frac{1000\times 0.7\times10^5}{200} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= 1\times10^5 \begin{bmatrix} 3.5 & -3.5 \\ -3.5 & 3.5 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

For element 2 : Nodes (2, 3)

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \frac{A_2 E_2}{l_2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

$$= 1\times10^5 \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

Assembling equation,

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = 1\times10^5 \begin{bmatrix} 3.5 & -3.5 & 0 \\ -3.5 & 13.5 & -10 \\ 0 & -10 & 10 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \longrightarrow (1)$$

Assembling of $\{F\}$ matrix

$$\{F\} = E A \alpha \Delta T \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$$

For element (1) $\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = E_1 A_1 \alpha_1 \Delta T \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$

$$= 0.7\times10^5 \times 1000 \times 23\times10^{-6} \times 30 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$$

$$= 1\times10^5 \begin{Bmatrix} -0.483 \\ 0.483 \end{Bmatrix}$$

For element (2) $\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = E_2 A_2 \alpha_2 \Delta T \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$

$$= 1\times10^5 \begin{Bmatrix} -1.08 \\ 1.08 \end{Bmatrix}$$

Assembling,

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = 1\times10^5 \begin{Bmatrix} -0.483 \\ 0.483 - 1.08 + 4 \\ 1.08 \end{Bmatrix} = 1\times10^5 \begin{Bmatrix} -0.483 \\ 3.403 \\ 1.08 \end{Bmatrix}$$

Applying boundary conditions in (1)

$u_1 = 0$, u_2, $u_3 = 0$,

$$1\times10^5 \begin{Bmatrix} -0.483 \\ 3.403 \\ 1.08 \end{Bmatrix} = 1\times10^5 \begin{bmatrix} 3.5 & -3.5 & 0 \\ -3.5 & 13.5 & -10 \\ 0 & -10 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ u_2 \\ 0 \end{bmatrix}$$

$$13.5\, u_2 = 1\times10^5 (3.403)$$

$$u_2 = 0.5251 \text{ mm},$$

Thermal stress,

$$\sigma = E \frac{du}{dx} - E\alpha\Delta T$$

$$\sigma_1 = E_1 \frac{(u_2 - u_1)}{l_1} - E_1 \alpha_1 \Delta T$$

$$= \frac{0.75\times10^5 \ (0.5251 - 0)}{200} - 0.75\times10^5 \times 23\times10^{-6} \times 30$$

$$\sigma_1 = 39.935 \text{ N/mm}^2 \ (\text{Tensile})$$

$$\sigma_2 = E_2 \frac{(u_3 - u_2)}{l_2} - E_2 \alpha_2 \Delta T$$

$$\sigma_2 = -240.066 \text{ N/mm}^2 \ [\text{compressive}].$$

To find (or) calculate the Reaction force.

$$\{R\} = [K]\{u^*\} - \{F\}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = 1\times10^5 \begin{bmatrix} 3.5 & -3.5 & 0 \\ -3.5 & 13.5 & -10 \\ 0 & -10 & 10 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.5251 \\ 0 \end{Bmatrix} - 1\times10^5 \begin{Bmatrix} -0.483 \\ 3.403 \\ 1.08 \end{Bmatrix}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = 1\times10^5 \begin{Bmatrix} -0.8823 \\ 3.905 \\ -2.521 \end{Bmatrix} - 1\times10^5 \begin{Bmatrix} -0.483 \\ 3.403 \\ 1.08 \end{Bmatrix}$$

$$= 1\times10^5 \begin{Bmatrix} -0.3993 \\ 0 \\ -3.601 \end{Bmatrix}$$

$R_1 = -0.3993 \times 10^5 \ N$

$R_2 = 0$

$R_3 = -3.601 \times 10^5 \ N$

verification,

$R_1 + R_2 + R_3 = -0.3993 \times 10^5 + 0 - 3.601 \times 10^5$

$= -4 \times 10^5 \ N$ (Applied force).

Hence its verified.