

**SNSCOLLEGEOFTECHNOLOGY** 

(AnAutonomousInstitution) Coimbatore-641035.



## UNIT 5- Laplace Transform

Inverse Laplace transforms

Inverse Laplace Transform: If the Laplace transform of f(t) is F(s) i.e., L[f(t)] = F(s). Then f(t) is called an inverse Laplace transform of F(s) and is written as  $f(t) = \Gamma'[F(s)]$  where  $\Gamma'$  is called the inverse Laplace transform Operator. Table of Inverse Laplace Transforms : .....  $(1 + [f(t)] = F(s) \qquad 1^{-1} [F(s)] = f(t)$  $1^{-1}\left(\frac{1}{5}\right) = 1$  $1 \quad L(1) = \frac{1}{S}$ (2)  $L(E) = \frac{1}{c^2} - \frac{1}{c^2} + \frac{1}{$ (3)  $L(t^n) = \frac{n!}{s^{n+1}} \qquad t^{-1} \left(\frac{n!}{s^{n+1}}\right) = t^n$  $-L^{-\prime}\left(\frac{i}{s-a}\right) = e^{at}$  $(4) \quad \bot (e^{\alpha t}) = \frac{1}{s-a}$  $L^{-1}\left(\frac{1}{s+a}\right) = e^{-at}$  $(E) \perp (e^{-\alpha t}) = \frac{1}{\hat{s} + \alpha}$ (b) L (sinat) =  $\frac{a}{S^2 + a^2}$  $L^{-1}\left(\frac{a}{s^{2}+a^{2}}\right) = sinat$  $L^{-1}\left(\frac{1}{s^2 + a^2}\right) = \frac{\sin at}{a}$  $(7) \perp \left(\frac{sinat}{a}\right) = \frac{1}{s^2 + a^2}$ (B)  $L(\cos at) = \frac{S}{S^2 + a^2}$  $L^{-1}\left(\frac{S}{C^2+q^2}\right) = \cos at$ (a)  $\perp (sinhat) = \frac{a}{s^2 - a^2}$  $L^{-1}\left(\frac{\alpha}{s^2 - \alpha^2}\right) = Sinhat$ (10)  $L(\cosh at) = \frac{S}{S^2 - a^2}$   $L^{-1}(\frac{S}{S^2 - a^2}) = \cosh at$  $(\Box) = \frac{1}{2} \left[ S(t) \right] = \frac{1}{2} \left[ 1 \right] \left[ 1 \right] = \frac{1}{2} \left[ 1 \right] = \frac{1}{2}$ 



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Problems : (1) Find  $L^{-1} \int \frac{S^2}{(S^2 + \sigma^2)(S^2 + h^2)}$ Soln:  $\frac{1}{1} \left[ \frac{S^2}{(S^2 + a^2)(S^2 + b^2)} \right] = \frac{1}{1} \left[ \frac{S^2 + a^2 - a^2}{(S^2 + a^2)(S^2 + b^2)} \right]$  $= \int_{-1}^{-1} \left[ \frac{1}{s^{2} + b^{2}} - \frac{a^{2}}{(s^{2} + a^{2})(s^{2} + b^{2})} \right]$  $= L^{-1} \left( \frac{1}{s^{2} + b^{2}} \right) = a^{2} L^{-1} \left( \frac{1}{(s^{2} + a^{2})(s^{2} + b^{2})} \right)$  $= \frac{1}{b} \frac{1}{b} \left( \frac{b}{s^2 + b^2} \right) - \frac{a^2}{b^2 - a^2} \frac{1}{b^2 - a^2} \left( \frac{b^2 - a^2}{(s^2 + a^2)(s^2 + b^2)} \right)$  $= \frac{1}{b} Sinbt - \frac{a^2}{b^2 - a^2} L^{-1} \left[ \frac{1}{S^2 + a^2} - \frac{1}{S^2 + b^2} \right]$  $= \frac{1}{b} \sin bt - \frac{a^2}{b^2 - a^2} \int \frac{1}{a} \sin at - \frac{1}{b} \sin bt \int \frac{1}{a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin at - \frac{1}{b^2 - a^2} \sin bt \int \frac{1}{a^2 - a^2} \sin$ (2) find  $\underline{\Gamma} \begin{bmatrix} \underline{2.5-5} \\ \underline{9.5^2-25} \end{bmatrix}$  $\frac{\mathcal{S}_{oh}}{L^{-1}\left(\frac{2S-5}{\alpha c^{3}-25}\right)} = L^{-1}\left[\frac{2S}{-9s^{2}-25} - \frac{5}{9s^{2}-25}\right]$  $= 1^{-1} \left[ \frac{2s}{9 \left[ s^2 - \left(\frac{5}{3}\right)^2 \right]} - \frac{5}{9 \left[ s^2 - \left(\frac{5}{3}\right)^2 \right]} \right]$  $= \frac{2}{9} \left[ \frac{5}{5^2 - \left(\frac{5}{3}\right)^2} - \frac{1}{3} L^{-1} \left[ \frac{5/3}{5^2 - \left(\frac{5}{3}\right)^2} \right] \right]$  $= \frac{2}{9} \cosh \frac{5}{3} t - \frac{1}{3} \sinh \frac{5}{3} t$ 



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(3) Find $L^{-1}\left[\frac{s}{(s+a)^2+4}\right]$ solutions	
$L^{-1}\left[\frac{S}{(S+2)^2+4}\right] = \frac{d}{dt}\left[L^{-1}\left(\frac{1}{(S+2)^2+4}\right)\right]$	]0
$= \frac{d}{dt} \left[ e^{-\lambda t} L^{-1} \left( \frac{1}{s^2 + \lambda^2} \right) \right]$ $= \frac{d}{dt} \left[ \frac{e^{-\lambda t}}{2} L^{-1} \left( \frac{2}{s^2 + \lambda^2} \right) \right]$	
$= \frac{d}{dt} \left( \frac{e^{-at}}{a} \sin at \right) = \frac{1}{2} \frac{d}{dt} \left( \frac{e^{-at}}{e} \sin at \right)$	)
$= \frac{1}{2} \left[ e^{-2t} \cos at(a) + \sin at(-a) e^{-at} \right]$	
$= \frac{1}{2} \left[ 2e^{-2t} \cos 2t - 2e^{-2t} \sin 2t \right]$	
$= e^{-at} (\cos at - \sin at)$	