Laplace transforms of derivatives:

If $L[f(t)] = F(s)$ then

$$L[f'(t)] = s\,F(s) - f(0).$$

Laplace Transform of integrals:

If $L[f(t)] = F(s)$ then $L\left[\int_0^t f(t)\,dt\right] = \frac{F(s)}{s}$

(1) Find $L[t \cos at]$

Soln:

$$\because L[t f(t)] = \frac{-d}{ds}[L(f(t))]$$

$$L[t \cos at] = \frac{-d}{ds}[L(\cos at)]$$

$$= \frac{-d}{ds}\left[\frac{s}{s^2+a^2}\right]$$

$$= -\left\{\frac{s^2+a^2-s(2s)}{(s^2+a^2)^2}\right\}$$

$$= -\left[\frac{a^2-s^2}{(s^2+a^2)^2}\right]$$

$$= \frac{s^2-a^2}{(s^2+a^2)^2}$$

(2) Find $L[te^{2t}\sin 3t]$

Soln:

$$L[te^{2t}\sin 3t] = \frac{-d}{ds}\{L[e^{2t}\sin 3t]\}$$

$$= \frac{-d}{ds}\{L(\sin 3t)\}_{s\to s-2}$$

$$= \frac{-d}{ds}\left\{\left(\frac{3}{s^2+9}\right)_{s\to s-2}\right\}$$

$$= -\left\{\frac{0-3(2s)}{(s^2+9)^2}\right\}_{s\to s-2}$$

$$= \left\{\frac{6s}{(s^2+9)^2}\right\}_{s\to s-2}$$

$$= \frac{6(s-2)}{[(s-2)^2+9]^2}$$

$$= \frac{6(s-2)}{(s^2-4s+13)^2}$$

(4) Find $L\left[\frac{\sin 3t}{t}\right]$

Soln:

$$L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)\,ds = \int_s^\infty L[f(t)]\,ds$$

$$L\left[\frac{\sin 3t}{t}\right] = \int_s^\infty L(\sin 3t)\,ds$$

$$= \int_s^\infty \left(\frac{3}{s^2+9}\right) ds$$

$$= \int_s^\infty \frac{3}{s^2+3^2}\,ds$$

$$= 3\cdot\frac{1}{3}\left[\tan^{-1}\left(\frac{s}{3}\right)\right]_s^\infty \qquad \left(\because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right)$$

$$= \tan^{-1}(\infty) - \tan^{-1}(s/3)$$

$$= \pi/2 - \tan^{-1}(s/3)$$

$$= \cot^{-1}(s/3).$$