



Laplace transforms of derivatives :

If  $L[f(t)] = F(s)$  then

$$L[f'(t)] = sF(s) - f(0).$$

Laplace Transform of integrals :

If  $L[f(t)] = F(s)$  then  $L \left[ \int_0^t f(t) dt \right] = \frac{F(s)}{s}$



# SNS COLLEGE OF TECHNOLOGY

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① Find  $L[t \cos at]$

Soln:

$$L[t f(t)] = -\frac{d}{ds} [L(f(t))]$$

$$L[t \cos at] = -\frac{d}{ds} [L(\cos at)]$$

$$= -\frac{d}{ds} \left[ \frac{s}{s^2 + a^2} \right]$$

$$= - \left\{ \frac{s^2 + a^2 - s(2s)}{(s^2 + a^2)^2} \right\}$$

$$= - \left[ \frac{a^2 - s^2}{(s^2 + a^2)^2} \right]$$

$$= \frac{s^2 - a^2}{(s^2 + a^2)^2}$$

② Find  $L[te^{2t} \sin 3t]$

Soln:

$$L[te^{2t} \sin 3t] = -\frac{d}{ds} \{ L[e^{2t} \sin 3t] \}$$

$$(s)^2 - (s)^2 + 2 = \left[ -\frac{d}{ds} \left\{ L(\sin 3t) \right\} \right]_{s \rightarrow s-2}$$

$$= -\frac{d}{ds} \left\{ \left( \frac{3}{s^2 + 9} \right) \right\}_{s \rightarrow s-2}$$

$$(3(s-2)) \lambda ds = \left\{ \frac{0 - 3(2s)(s^2 + 9)}{(s^2 + 9)^2} \right\}_{s \rightarrow s-2}$$

$$\frac{2 \cdot 3s}{(s^2 + 9)^2} = \left\{ \frac{6s}{(s^2 + 9)^2} \right\}_{s \rightarrow s-2}$$

$$\frac{2s^3 + 18s}{(s^2 + 9)^3} = \left\{ \frac{6s}{(s^2 + 9)^2} \right\}_{s \rightarrow s-2}$$



$$= \frac{6(s-2)}{[(s-2)^2 + 9]^2}$$

$$= \frac{6(s-2)}{(s^2 - 4s + 13)^2}$$

(4) Find  $\mathcal{L} \left[ \frac{\sin 3t}{t} \right]$

Soln:  $\mathcal{L} \left[ \frac{f(t)}{t} \right] = \int_s^\infty f(s) ds = \int_{s+\alpha}^\infty f(t) dt$

$$\mathcal{L} \left[ \frac{\sin 3t}{t} \right] = \int_s^\infty \mathcal{L}(\sin 3t) ds$$

$$= \int_s^\infty \left( \frac{3}{s^2 + 9} \right) ds$$

$$= \int_s^\infty \frac{3}{s^2 + 3^2} ds$$

$$= 3 \cdot \frac{1}{3} \left[ \tan^{-1} \left( \frac{s}{3} \right) \right]_s^\infty \quad (\because \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right))$$

$$= \tan^{-1}(\infty) - \tan^{-1}(s/3) = \frac{1}{3} \tan^{-1} \left( \frac{s}{3} \right)$$

$$= \pi/2 - \tan^{-1}(s/3)$$

$$= \cot^{-1}(s/3).$$