Unit-V

Laplace Transforms

Introduction:

Laplace transformation, named after a great French Mathematician Pierre Simon De Laplace (1749-1827) who used such transformation in the "Theory of Probability".

Uses of Laplace Transformation:

1. It is used to find the solution of linear differential equations Ordinary as well as Partial.
2. It helps in solving the differential equation with boundary Values without finding the general solution and then finding the values of the arbitrary constants.

Transformation:

A transformation is an operation which converts a mathematical expression to a different but equivalent form.

Laplace Transformation: Definition

Let $f(t)$ be a function of t defined for $t>0$ then the laplace transform of $f(t)$, denoted by $L\{f(t)\}$ or $F(s)$ is defined by,

$$L[f(t)] = \int_0^\infty e^{-st} f(t)dt = F(s)$$

Provided the integral exists.

Conditions for existence of Laplace transform:

i) $f(t)$ should be continuous or piecewise continuous in the given closed interval $[a,b]$ where $a>0$.

ii) $f(t)$ should be of exponential order.

Exponential order:

A function $f(t)$ is said to be of exponential order if

$$\lim_{t\to\infty} e^{-st} f(t) = 0.$$

Example:

i) t^2 is of exponential order.

$$\lim_{t\to\infty} e^{-st} f(t) = \lim_{t\to\infty} e^{-st} t^2$$

$$= \lim_{t\to\infty} \frac{t^2}{e^{tst}} \quad \left(\frac{\infty}{\infty} \text{ indeterminant form}\right)$$

$$= \lim_{t\to\infty} \frac{2t}{se^{tst}}$$ Apply L'Hospital rule

$$= \lim_{t\to\infty} \frac{2}{s^2 e^{st}}$$

$$= \frac{2}{\infty}$$

$$= 0$$

ii) e^{t^2} is not of exponential order.

$$\lim_{t\to\infty} e^{-st} f(t) = \lim_{t\to\infty} e^{-st} e^{t^2}$$

$$= \lim_{t\to\infty} e^{-st+t^2}$$

$= e^{\infty}$

$= \infty$

$\therefore e^{t^2}$ is not of exponential order.