



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

Transforms of elementary functions:

1) $L(1) = \frac{1}{s}$ where $s > 0$.

Proof:

$$L\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$$

$$L(1) = \int_0^{\infty} e^{-st} \cdot 1 dt$$

$$= \left[\frac{e^{-st}}{-s} \right]_0^{\infty}$$

$$= -\frac{1}{s} (0 - 1) = \frac{1}{s}$$

$$L(1) = \frac{1}{s}$$

2) $L(K) = \frac{K}{s}$.

3) $L(t) = \frac{1!}{s^2}$

$$L(t) = \int_0^{\infty} e^{-st} \cdot t dt$$

$$= \left[\frac{te^{-st}}{-s} - \frac{e^{-st}}{s^2} \right]_0^{\infty}$$

$$L(t) = \frac{1!}{s^2}$$

Bernoulli's formula:

$$I = uv_1 - u'v_2 + u''v_3$$

$$\begin{cases} u = t \\ u' = 1 \\ u'' = 0 \end{cases} \quad \begin{cases} v = e^{-st} \\ v' = -e^{-st} \\ v_2 = \frac{e^{-st}}{s^2} \end{cases}$$



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$$4) L(t^2) = \frac{2!}{s^3}$$

$$5) L(t^n) = \frac{n!}{s^{n+1}} \text{ if } s > 0 \text{ and } n > -1$$

$$L(t^n) = \int_0^{\infty} e^{-st} \cdot t^n dt$$

$$\text{Put } x = st, \quad dx = s dt$$

$$\frac{dx}{s} = dt$$

$$L(t^n) = \int_0^{\infty} e^{-x} \left(\frac{x}{s}\right)^n \frac{dx}{s}$$

$$= \int_0^{\infty} e^{-x} \cdot \frac{x^n}{s^{n+1}} dx$$

$$= \frac{1}{s^{n+1}} \int_0^{\infty} e^{-x} x^n dx$$

$$L(t^n) = \frac{n!}{s^{n+1}} = \frac{n!}{s^{n+1}}$$

$$6) L(e^{at}) = \frac{1}{s-a} \text{ if } s-a > 0.$$

$$L(e^{at}) = \int_0^{\infty} e^{-st} e^{at} dt$$

$$= \int_0^{\infty} e^{-(s-a)t} dt$$

$$= \left[\frac{e^{-(s-a)t}}{-(s-a)} \right]_0^{\infty}$$



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$$L(e^{at}) = \frac{1}{s-a} \quad \text{if } s-a > 0.$$

$$7) L(e^{-at}) = \frac{1}{s+a} \quad \text{if } s+a > 0.$$

$$\begin{aligned} L(e^{-at}) &= \int_0^{\infty} e^{-st} e^{-at} dt \\ &= \int_0^{\infty} e^{-(s+a)t} dt \end{aligned}$$

$$L(e^{-at}) = \frac{1}{s+a} \quad \text{if } s+a > 0.$$

8) To find $L(\cos at)$ and $L(\sin at)$.

We know $e^{i\theta} = \cos\theta + i\sin\theta$.

$$\begin{aligned} L(e^{iat}) &= \frac{1}{s-ia} \\ &= \frac{1}{s-ia} \cdot \frac{s+ia}{s+ia} \\ &= \frac{s+ia}{s^2+a^2} \end{aligned}$$

$$L(\cos at + i\sin at) = \frac{s}{s^2+a^2} + i \frac{a}{s^2+a^2}$$

Equating real & imaginary Parts,

$$L(\cos at) = \frac{s}{s^2+a^2}$$

$$L(\sin at) = \frac{a}{s^2+a^2}$$



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9) To find $L(\sinh at) \therefore$

$$L[\sinh at] = L\left(\frac{e^{at} - e^{-at}}{2}\right)$$

$$= \frac{1}{2} L(e^{at}) - \frac{1}{2} L(e^{-at})$$

$$= \frac{1}{2} \left(\frac{1}{s-a} - \frac{1}{s+a} \right)$$

$$= \frac{1}{2} \left(\frac{2a}{s^2 - a^2} \right)$$

$$L(\sinh at) = \frac{a}{s^2 - a^2} \quad \text{for } s > a$$

10) To find $L(\cosh at) \therefore$

$$L(\cosh at) = L\left(\frac{1}{2} (e^{at} + e^{-at})\right)$$

$$= \frac{1}{2} L(e^{at}) + \frac{1}{2} L(e^{-at})$$

$$= \frac{1}{2} \left(\frac{1}{s-a} + \frac{1}{s+a} \right)$$

$$= \frac{1}{2} \frac{2s}{s^2 - a^2}$$

$$L(\cosh at) = \frac{s}{s^2 - a^2} \quad \text{for } s > a$$

Problems:

1) Find $L(t^8)$.

Sol:

$$L(t^n) = \frac{n!}{s^{n+1}}$$

$$L(t^8) = \frac{8!}{s^{8+1}} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{s^9} = \frac{40320}{s^9}$$



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2) Find $L(t+1)$.

$$\begin{aligned}\text{sol: } L(t+1)^2 &= L(t^2 + 2t + 1) \\ &= L(t^2) + 2L(t) + L(1) \\ &= \frac{2!}{s^3} + \frac{2}{s^2} + \frac{1}{s}\end{aligned}$$

3) Find $L\left(\frac{1}{\sqrt{t}}\right)$.

$$\begin{aligned}\text{sol: } L\left(\frac{1}{\sqrt{t}}\right) &= L(t^{-1/2}) \\ &= \frac{\Gamma(-\frac{1}{2}+1)}{s^{-1/2+1}} = \frac{\Gamma(1/2)}{s^{1/2}} = \frac{\sqrt{\pi}}{\sqrt{s}}\end{aligned}$$

4) $L(\sqrt{t})$:-

$$\begin{aligned}\text{sol: } L(\sqrt{t}) &= L(t^{1/2}) \\ &= \frac{\Gamma(1/2+1)}{s^{1/2+1}} = \frac{\frac{1}{2}\Gamma(1/2)}{s^{3/2}} = \frac{\frac{1}{2}\sqrt{\pi}}{s^{3/2}} \\ &= \frac{\sqrt{\pi}}{2s^{3/2}}\end{aligned}$$

$$\boxed{\begin{aligned}\Gamma(n+1) &= n\Gamma(n) \\ \Gamma(1/2) &= \sqrt{\pi}\end{aligned}}$$

5) $L(t^{5/2})$.

$$\begin{aligned}\text{sol: } L(t^{5/2}) &= \frac{\Gamma(5/2+1)}{s^{5/2+1}} = \frac{\frac{5}{2}\Gamma(5/2)}{s^{7/2}} \\ &= \frac{\frac{5}{2} \cdot \frac{3}{2} \cdot \frac{1}{2}\Gamma(1/2)}{s^{7/2}} \\ &= \frac{15\sqrt{\pi}}{8s^{7/2}}\end{aligned}$$



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6) $L(e^{5t})$.

Sol. $L(e^{at}) = \frac{1}{s-a}$

$$L(e^{5t}) = \frac{1}{s-5}$$

7) $L(e^t)$.

Sol. $L(e^t) = \frac{1}{s-1}$

8) $L(e^{-7t})$.

Sol. $L(e^{-7t}) = \frac{1}{s-(-7)} = \frac{1}{s+7}$

9) $L(e^{-t})$.

Sol. $L(e^{-t}) = \frac{1}{s-(-1)} = \frac{1}{s+1}$

10) Find $L(\sin 5t)$

$$L(\sin at) = \frac{a}{s^2+a^2}$$

$$L(\sin 5t) = \frac{5}{s^2+5^2} = \frac{5}{s^2+25}$$

11) Find $L(\cos 6t)$.

$$L(\cos at) = \frac{s}{s^2+a^2}$$

$$\begin{aligned} L(\cos 6t) &= \frac{s}{s^2+6^2} \\ &= \frac{s}{s^2+36} \end{aligned}$$



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12) Find $L(\sin^2 t)$.

Sol. $\sin^2 t = \frac{1 - \cos 2t}{2}$

$$L(\sin^2 t) = L\left(\frac{1 - \cos 2t}{2}\right)$$

$$= \frac{1}{2} L(1 - \cos 4t)$$

$$= \frac{1}{2} [L(1) - L(\cos 4t)]$$

$$= \frac{1}{2} \left[\frac{1}{s} - \frac{s}{s^2 + 16} \right]$$

13) Find $L(\cos^2 t)$.

Sol. $\cos^2 t = \frac{1 + \cos 2t}{2}$

$$L(\cos^2 t) = L\left(\frac{1 + \cos 2t}{2}\right)$$

$$= \frac{1}{2} L(1 + \cos 4t)$$

$$= \frac{1}{2} [L(1) + L(\cos 4t)]$$

$$= \frac{1}{2} \left(\frac{1}{s} + \frac{s}{s^2 + 16} \right)$$

14) Find $L(\cos^3 t)$.

Sol. $\cos^3 t = \frac{1}{4} (\cos 3t + 3 \cos t)$

$$L(\cos^3 t) = L\left(\frac{\cos 3t + 3 \cos t}{4}\right)$$

$$= \frac{1}{4} \{L(\cos 3t) + 3L(\cos t)\}$$

$$= \frac{1}{4} \left\{ \frac{s}{s^2 + 9} + 3 \frac{s}{s^2 + 1} \right\}$$



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$$= \frac{1}{4} \left\{ \frac{s}{s^2+36} + 3 \frac{s}{s^2+4} \right\}$$

$$= \frac{1}{4} \left\{ \frac{s}{s^2+36} + \frac{3s}{s^2+4} \right\}$$

15) Find $L(\sin^3 3t)$.

Sol:- $\sin^3 \theta = \frac{3 \sin \theta - \sin 3\theta}{4}$

$$= L \left(\frac{3 \sin 3t - \sin 9t}{4} \right)$$

$$= \frac{1}{4} \left\{ 3L(\sin 3t) - L(\sin 9t) \right\}$$

$$= \frac{1}{4} \left\{ 3 \left(\frac{3}{s^2+3^2} \right) - \frac{9}{s^2+9^2} \right\}$$

$$= \frac{9}{4} \left\{ \frac{1}{s^2+9} - \frac{1}{s^2+81} \right\}$$

16) Find $L(\sin 2t \cos 3t)$

Sol:- $\sin A \cos B = \frac{\sin(A+B) + \sin(A-B)}{2}$

$$L(\sin 2t \cos 3t) = L \left\{ \frac{\sin(2t+3t) + \sin(2t-3t)}{2} \right\}$$

$$= \frac{1}{2} \left\{ L(\sin 5t) + L(\sin(-t)) \right\}$$

$$= \frac{1}{2} \left\{ L(\sin 5t) - L(\sin t) \right\}$$

$$= \frac{1}{2} \left\{ \frac{5}{s^2+25} - \frac{1}{s^2+1} \right\}$$