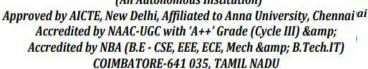
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Applications of Laplace transforms to Differential Equations.

If
$$L[f(t)] = F(s)$$
 then

 $L[y'(t)] = SL(y) - y(0)$
 $L[y''(t)] = S^2L(y) - Sy(0) - y'(0)$.

I) Solve the differential equations using Laplace transforms

 $L[y''(t)] = S^2L(y) - Sy(0) - y'(0) = 0$.

Sol:.
$$y'' + 4y' + 4y = e^{-t}$$

Taking Laplace transforms we get,

 $L(y'' + 4y' + 4y) = L(e^{-t})$
 $L(y'') + 4L(y') + 4L(y) = \frac{1}{s+1}$
 $[s^2L(y) - sy(0) - y'(0)] + 4[sL(y) - y(0)] + 4L(y) = \frac{1}{s+1}$
 $[s^2L(y) - sx0 - 0] + 4[sL(y) - 0] + 4L(y) = \frac{1}{s+1}$
 $[s^2L(y) + 4sL(y) + 4L(y) = \frac{1}{s+1}$
 $(s^2 + 4s + 4)L(y) = \frac{1}{s+1}$
 $L(y)(s + 2)^2 = \frac{1}{s+1}$

 $L(y) = \frac{1}{(c+1)(s+2)^2}$

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$$y = L^{-1} \left[\frac{1}{(s+1)(s+2)^{2}} \right]$$

$$\frac{1}{(s+1)[s+2)^{2}} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^{2}}$$

$$1 = A(s+2)^{2} + B(s+2)(s+1) + C(s+1)$$

$$S = -2 \Rightarrow C = -1$$

$$S = -1 \Rightarrow A = 1$$

$$S = 0 \Rightarrow B = -1$$

$$\frac{1}{(s+1)(s+2)^{2}} = \frac{1}{s+1} - \frac{1}{s+2} - \frac{1}{(s+2)^{2}}$$

$$= L^{-1} \left(\frac{1}{(s+1)(s+2)^{2}} \right)$$

$$= L^{-1} \left(\frac{1}{(s+1)} - L^{-1} \left(\frac{1}{s+2} \right) - L^{-1} \left(\frac{1}{(s+2)^{2}} \right)$$

$$= e^{-t} - e^{-2t} + e^{-2t}$$

$$= e^{-t} L^{-1} \left(\frac{1}{s^{2}} \right)$$

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