

Transforms of elementary functions:

$$1) L(1) = \frac{1}{s} \text{ where } s > 0.$$

Proof:-

$$L\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$$

$$L(1) = \int_0^{\infty} e^{-st} \cdot 1 dt$$

$$= \left[\frac{e^{-st}}{-s} \right]_0^{\infty}$$

$$= -\frac{1}{s} (0 - 1) = \frac{1}{s}$$

$$L(1) = \frac{1}{s}$$

$$2) L(K) = \frac{K}{s}.$$

$$3) L(t) = \frac{1!}{s^2}$$

$$L(t) = \int_0^{\infty} e^{-st} \cdot t dt$$

$$= \left[\frac{t e^{-st}}{-s} - \frac{e^{-st}}{s^2} \right]_0^{\infty}$$

$$L(t) = \frac{1!}{s^2}$$

Bernoulli's formula:-

$$I = uv_1 - u'v_2 + u''v_3$$

$$\begin{aligned} u &= t & \begin{cases} v = e^{-st} \\ v' = e^{-st} \\ v_2 = \frac{e^{-st}}{s^2} \end{cases} \\ u' &= 1 \\ u'' &= 0 \end{aligned}$$



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$$4) L(t^2) = \frac{2!}{s^3}$$

$$5) L(t^n) = \frac{n!}{s^{n+1}} \text{ if } s > 0 \text{ and } n > -1$$

$$L(t^n) = \int_0^{\infty} e^{-st} \cdot t^n dt$$

$$\text{Put } x = st, dx = s dt$$

$$\frac{dx}{s} = dt$$

$$L(t^n) = \int_0^{\infty} e^{-x} \left(\frac{x}{s}\right)^n \frac{dx}{s}$$

$$= \int_0^{\infty} e^{-x} \cdot \frac{x^n}{s^{n+1}} dx$$

$$= \frac{1}{s^{n+1}} \int_0^{\infty} e^{-x} x^n dx$$

$$L(t^n) = \frac{n!}{s^{n+1}} = \frac{n!}{s^{n+1}}$$

$$6) L(e^{at}) = \frac{1}{s-a} \text{ if } s-a > 0.$$

$$L(e^{at}) = \int_0^{\infty} e^{-st} e^{at} dt$$

$$= \int_0^{\infty} e^{-(s-a)t} dt$$

$$= \left[\frac{e^{-(s-a)t}}{-(s-a)} \right]_0^{\infty}$$



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$$L(e^{at}) = \frac{1}{s-a} \quad \text{if } s-a > 0.$$

$$7) L(e^{-at}) = \frac{1}{s+a} \quad \text{if } s+a > 0.$$

$$\begin{aligned} L(e^{-at}) &= \int_0^{\infty} e^{-st} e^{-at} dt \\ &= \int_0^{\infty} e^{-(s+a)t} dt \end{aligned}$$

$$L(e^{-at}) = \frac{1}{s+a} \quad \text{if } s+a > 0.$$

8) To find $L(\cos at)$ and $L(\sin at)$.

We know $e^{i\theta} = \cos \theta + i \sin \theta$.

$$\begin{aligned} L(e^{iat}) &= \frac{1}{s-ia} \\ &= \frac{1}{s-ia} \cdot \frac{s+ia}{s+ia} \\ &= \frac{s+ia}{s^2+a^2} \end{aligned}$$

$$L(\cos at + i \sin at) = \frac{s}{s^2+a^2} + i \frac{a}{s^2+a^2}$$

Equating real & imaginary parts,

$$L(\cos at) = \frac{s}{s^2+a^2}$$

$$L(\sin at) = \frac{a}{s^2+a^2}$$



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9) To find $L(\sinh at) :-$

$$L[\sinh at] = L\left(\frac{e^{at} - e^{-at}}{2}\right)$$

$$= \frac{1}{2} L(e^{at}) - \frac{1}{2} L(e^{-at})$$

$$= \frac{1}{2} \left(\frac{1}{s-a} - \frac{1}{s+a} \right)$$

$$= \frac{1}{2} \left(\frac{2a}{s^2 - a^2} \right)$$

$$L(\sinh at) = \frac{a}{s^2 - a^2} \quad \text{for } s^2 > a^2$$

10) To find $L(\cosh at) :-$

$$L(\cosh at) = L\left(\frac{1}{2} (e^{at} + e^{-at})\right)$$

$$= \frac{1}{2} L(e^{at}) + \frac{1}{2} L(e^{-at})$$

$$= \frac{1}{2} \left(\frac{1}{s-a} + \frac{1}{s+a} \right)$$

$$= \frac{1}{2} \frac{2s}{s^2 - a^2}$$

$$L(\cosh at) = \frac{s}{s^2 - a^2} \quad \text{for } s^2 > a^2$$

Problems:-

1) Find $L(t^8)$.

Sol:- $L(t^n) = \frac{n!}{s^{n+1}}$

$$L(t^8) = \frac{8!}{s^{8+1}} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{s^9} = \frac{40320}{s^9}$$



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2) Find $L(t+1)$.

$$\begin{aligned}\text{sol: } L(t+1)^2 &= L(t^2 + 2t + 1) \\ &= L(t^2) + 2L(t) + L(1) \\ &= \frac{2!}{s^3} + \frac{2}{s^2} + \frac{1}{s}\end{aligned}$$

3) Find $L\left(\frac{1}{\sqrt{t}}\right)$.

$$\begin{aligned}\text{sol: } L\left(\frac{1}{\sqrt{t}}\right) &= L(t^{-1/2}) \\ &= \frac{\Gamma(-\frac{1}{2}+1)}{s^{-1/2+1}} = \frac{\Gamma^{1/2}}{s^{1/2}} = \frac{\sqrt{\pi}}{\sqrt{s}}\end{aligned}$$

4) $L(\sqrt{t})$:-

$$\begin{aligned}\text{sol: } L(\sqrt{t}) &= L(t^{1/2}) \\ &= \frac{\Gamma^{1/2+1}}{s^{1/2+1}} = \frac{\frac{1}{2} \Gamma^{1/2}}{s^{3/2}} = \frac{\frac{1}{2} \sqrt{\pi}}{s^{3/2}} \\ &= \frac{\sqrt{\pi}}{2s^{3/2}}\end{aligned}$$

$$\begin{aligned}\Gamma^{n+1} &= n \Gamma^n \\ \Gamma^{1/2} &= \sqrt{\pi}\end{aligned}$$

5) $L(t^{5/2})$.

$$\begin{aligned}\text{sol: } L(t^{5/2}) &= \frac{\Gamma^{5/2+1}}{s^{5/2+1}} = \frac{\frac{5}{2} \Gamma^{5/2}}{s^{7/2}} \\ &= \frac{\frac{5}{2} \cdot \frac{3}{2} \cdot \frac{1}{2} \Gamma^{1/2}}{s^{7/2}} \\ &= \frac{15 \sqrt{\pi}}{8s^{7/2}}\end{aligned}$$



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12) Find $L(\sin^2 2t)$.

Sol: $\sin^2 2t = \frac{1 - \cos 4t}{2}$

$$L(\sin^2 2t) = L\left(\frac{1 - \cos 4t}{2}\right)$$

$$= \frac{1}{2} L(1 - \cos 4t)$$

$$= \frac{1}{2} [L(1) - L(\cos 4t)]$$

$$= \frac{1}{2} \left[\frac{1}{s} - \frac{s}{s^2 + 16} \right]$$

13) Find $L(\cos^2 3t)$.

Sol: $\cos^2 t = \frac{1 + \cos 2t}{2}$

$$L(\cos^2 3t) = L\left(\frac{1 + \cos 2(3t)}{2}\right)$$

$$= \frac{1}{2} L(1 + \cos 6t)$$

$$= \frac{1}{2} [L(1) + L(\cos 6t)]$$

$$= \frac{1}{2} \left(\frac{1}{s} + \frac{s}{s^2 + 36} \right)$$

14) Find $L(\cos^3 2t)$.

Sol: $\cos^3 \theta = \frac{1}{4} (\cos 3\theta + 3\cos \theta)$

$$L(\cos^3 2t) = L\left(\frac{\cos 3(2t) + 3\cos 2t}{4}\right)$$

$$= \frac{1}{4} \{L(\cos 6t) + 3L(\cos 2t)\}$$

$$= \frac{1}{4} \left\{ \frac{s}{s^2 + 36} + 3 \frac{s}{s^2 + 4} \right\}$$