



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)



Properties:

change of Scale Property ..

If $L\{f(t)\} = F(s)$, then

$$L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right).$$

Proof:

We know that,

$$L[f(at)] = \int_0^{\infty} e^{-st} f(at) dt$$

Put $at = x$

$$t = \frac{x}{a} \quad dt = \frac{dx}{a}$$

$$L[f(at)] = \int_0^{\infty} e^{-s(x/a)} f(x) \frac{dx}{a}$$

$$= \frac{1}{a} \int_0^{\infty} e^{-(s/a)x} f(x) dx$$

$$= \frac{1}{a} \int_0^{\infty} e^{-(s/a)t} f(t) dt$$

$$L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$$

First shifting Property:

If $L\{f(t)\} = F(s)$ then

$$i) L[e^{-at} f(t)] = \{L[f(t)]\}_{s \rightarrow s+a} = F(s+a)$$

$$ii) L[e^{at} f(t)] = \{L[f(t)]\}_{s \rightarrow s-a} = F(s-a)$$

Proof:



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i) We know that,

$$L[f(t)] = \int_0^{\infty} e^{-st} f(t) dt = F(s)$$

$$\begin{aligned} L[e^{-at} f(t)] &= \int_0^{\infty} e^{-st} [e^{-at} f(t)] dt \\ &= \int_0^{\infty} e^{-(s+a)t} f(t) dt \\ &= F(s+a) \end{aligned}$$

$$\begin{aligned} \text{ii) } L(e^{at} f(t)) &= \int_0^{\infty} e^{-st} [e^{at} f(t)] dt \\ &= \int_0^{\infty} e^{-(s-a)t} f(t) dt \\ &= F(s-a) \end{aligned}$$

Second shifting Property

$$\text{If } L\{f(t)\} = F(s) \text{ and } g(t) = \begin{cases} f(t-a), & t > a \\ 0, & t < a \end{cases}$$

$$\text{then } L[g(t)] = e^{-as} F(s).$$

Proof:

$$L[g(t)] = \int_0^{\infty} e^{-st} g(t) dt$$

$$= \int_0^a e^{-st} g(t) dt + \int_a^{\infty} e^{-st} g(t) dt$$

$$L[g(t)] = 0 + \int_a^{\infty} e^{-st} f(t-a) dt$$



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$$= \int_a^{\infty} e^{-st} f(t-a) dt$$

$$\text{Put } t-a=u \Rightarrow dt=du$$

$$\text{when } t=a \Rightarrow u=0$$

$$t=\infty \Rightarrow u=\infty$$

$$L[g(t)] = \int_0^{\infty} e^{-s(u+a)} f(u) du$$

$$= \int_0^{\infty} e^{-us} \cdot e^{-as} f(u) du$$

$$= e^{-as} \int_0^{\infty} e^{-us} f(u) du$$

$$= e^{-as} \int_0^{\infty} e^{-st} f(t) dt$$

Replace $u \rightarrow t$

$$= e^{-as} F(s)$$

$$L[g(t)] = e^{-as} F(s)$$

Problems on change of Scale Property:-

1) Find $L(\sinh 3t)$ by using change of scale Property.

$$\text{Sol: } L(\sinh t) = \frac{1}{s^2-1} = F(s)$$

$$L(\sinh 3t) = \frac{1}{3} F\left(\frac{s}{3}\right)$$

$$= \frac{1}{3} \frac{1}{\left(\frac{s}{3}\right)^2 - 1}$$

$$= \frac{1}{3} \frac{1}{\left(\frac{s}{3}\right)^2 - 1}$$



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$$= \frac{1}{3} \left(\frac{9}{s^2 - 9} \right)$$

$$= \frac{3}{s^2 - 9}$$

$$L(\sinh 3t) = \frac{3}{s^2 - 9}$$

2) Find $L(\cos 5t)$ using change of scale Property:

Sol: $L(\cos t) = \frac{s}{s^2 + 1} = F(s)$

$$L(\cos 5t) = \frac{1}{5} F\left(\frac{s}{5}\right)$$

$$= \frac{1}{5} \left[\frac{s/5}{\left(\frac{s}{5}\right)^2 + 1} \right]$$

$$= \frac{1}{5} \left[\frac{5s}{s^2 + 25} \right]$$

$$L(\cos 5t) = \frac{s}{s^2 + 25}$$

3) Given $L[f(t)] = \frac{s^2 - s + 1}{(2s+1)^2(s-1)}$ applying the change of

Scale Property show that $L[f(2t)] = \frac{s^2 - 2s + 4}{4(s+1)^2(s-2)}$

Sol: $L[f(t)] = \frac{s^2 - s + 1}{(2s+1)^2(s-1)} = F(s)$

$$L[f(2t)] = \frac{1}{2} F\left(\frac{s}{2}\right)$$



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$$= \frac{1}{2} \left[\frac{\left(\frac{s}{2}\right)^2 - \left(\frac{s}{2}\right) + 1}{\left(2\frac{s}{2} + 1\right)^2 \left(\frac{s}{2} - 1\right)} \right]$$

$$= \frac{1}{2} \left[\frac{\frac{s^2 - 2s + 4}{4}}{\frac{4}{(s+1)^2(s-2)/2}} \right]$$

$$= \frac{1}{4} \frac{s^2 - 2s + 4}{(s+1)^2(s-2)}$$

$$\boxed{L[f(2t)] = \frac{s^2 - 2s + 4}{4(s+1)^2(s-2)}}$$

4) Find $L(e^{5t})$ applying change of scale property.

Sol: $L(e^t) = \frac{1}{s-1} = F(s)$

$$L(e^{5t}) = \frac{1}{5} F\left(\frac{s}{5}\right)$$

$$= \frac{1}{5} \frac{1}{\frac{s}{5} - 1}$$

$$= \frac{1}{5} \frac{5}{s-5}$$

$$= \frac{1}{s-5}$$

$$L(e^{5t}) = \frac{1}{s-5}$$