



# SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)



$$\begin{aligned}
&= \int_s^{\infty} \frac{3}{s^2 + 3^2} ds \\
&= 3 \cdot \frac{1}{3} \left[ \tan^{-1} \left( \frac{s}{3} \right) \right]_s^{\infty} \\
&= \tan^{-1}(\infty) - \tan^{-1} \left( \frac{s}{3} \right) \\
&= \frac{\pi}{2} - \tan^{-1} \left( \frac{s}{3} \right) \\
&= \cot^{-1} \left( \frac{s}{3} \right)
\end{aligned}$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right)$$

Integral of Laplace Transform (or) Laplace transform of

$$\frac{f(t)}{t}$$

If  $L[f(t)] = F(s)$  and if  $\lim_{t \rightarrow 0} \frac{f(t)}{t}$  exists then

$$L \left[ \frac{f(t)}{t} \right] = \int_s^{\infty} F(s) ds$$

Proof:

$$L[f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t) dt$$

Integrating w.r to 's' from s to  $\infty$ , we get

$$\int_s^{\infty} F(s) ds = \int_s^{\infty} \left[ \int_0^{\infty} e^{-st} f(t) dt \right] ds$$

$$= \int_0^{\infty} \left[ \int_s^{\infty} e^{-st} f(t) ds \right] dt$$

$$= \int_0^{\infty} f(t) \left[ \int_s^{\infty} e^{-st} ds \right] dt$$



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$$= \int_0^{\infty} f(t) \left[ \frac{e^{-st}}{-s} \right]_s^{\infty} dt$$

$$= \int_0^{\infty} f(t) \left[ 0 - \frac{e^{-st}}{-s} \right] dt$$

$$= \int_0^{\infty} e^{-st} \frac{f(t)}{t} dt$$

$$= L \left[ \frac{f(t)}{t} \right]$$

$$L \left[ \frac{f(t)}{t} \right] = \int_s^{\infty} F(s) ds$$

Problems:-

1) Find  $L \left( \frac{1-\cos t}{t} \right)$

Sol:-  $L \left( \frac{1-\cos t}{t} \right) = \int_s^{\infty} L(1-\cos t) ds$

$$= \int_s^{\infty} \{ L(1) - L(\cos t) \} ds$$

$$= \int_s^{\infty} \left\{ \frac{1}{s} - \frac{s}{s^2+1} \right\} ds$$

$$= \left( \log s - \frac{1}{2} \log (s^2+1) \right)_s^{\infty}$$

$$= \left( \log s - \log (s^2+1)^{1/2} \right)_s^{\infty}$$



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$$= \left[ \log \frac{s}{\sqrt{s^2+1}} \right]_s^\infty$$

$$= \left[ \log \frac{1}{\sqrt{1+\frac{1}{s^2}}} \right]_s^\infty$$

$$= \log 1 - \log \left[ \frac{1}{\sqrt{1+\frac{1}{s^2}}} \right]$$

$$= 0 - \log \frac{s}{\sqrt{s^2+1}}$$

$$= \log \left[ \frac{s}{\sqrt{s^2+1}} \right]^{-1}$$

$$= \log \left( \frac{\sqrt{s^2+1}}{s} \right)$$

$$L\left(\frac{1-\cos t}{t}\right) = \log \left( \frac{\sqrt{s^2+1}}{s} \right)$$

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2) Find  $L\left(\frac{e^{-3t}-e^{-4t}}{t}\right)$

Sol:  $L(e^{-3t}-e^{-4t}) = \frac{1}{s+3} - \frac{1}{s+4}$

$$L\left(\frac{e^{-3t}-e^{-4t}}{t}\right) = \int_s^\infty \left( \frac{1}{s+3} - \frac{1}{s+4} \right) ds$$

$$= \int_s^\infty \left( \frac{1}{s+3} - \frac{1}{s+4} \right) ds$$

$$= \left[ \log(s+3) - \log(s+4) \right]_s^\infty$$

$$= \left( \log \left( \frac{s+3}{s+4} \right) \right)_s^\infty$$



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$$= \left[ \log \left( \frac{s+3}{s+4} \right) \right]_s^{\infty}$$

$$= \log \left( \frac{s+4}{s+3} \right).$$

3) Find  $L \left( \frac{1 - \cos at}{t} \right)$

Sol.  $L \left( \frac{1 - \cos at}{t} \right) = \int_s^{\infty} L(1 - \cos at) ds$

$$= \int_s^{\infty} \left( \frac{1}{s} - \frac{s}{a^2 + s^2} \right) ds$$

$$= \left( \log s - \frac{1}{2} \log(s^2 + a^2) \right) \Big|_s^{\infty}$$

$$= \left( \log \left( \frac{s}{\sqrt{s^2 + a^2}} \right) \right) \Big|_s^{\infty}$$

$$= 0 - \log \left( \frac{s}{\sqrt{s^2 + a^2}} \right)$$

$$= \log \left( \frac{\sqrt{s^2 + a^2}}{s} \right)$$

$$L \left( \frac{1 - \cos at}{t} \right) = \log \left( \frac{\sqrt{s^2 + a^2}}{s} \right)$$

4) Find  $L \left( \frac{\cos at - \cos bt}{t} \right)$

Sol.  $L \left( \frac{\cos at - \cos bt}{t} \right) = \int_s^{\infty} L(\cos at - \cos bt) ds$





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$$\begin{aligned} &= \int_s^{\infty} \left( \frac{a}{s^2+a^2} - \frac{b}{s^2+b^2} \right) ds \\ &= \frac{1}{2} \left[ \log(s^2+a^2) - \log(s^2+b^2) \right]_s^{\infty} \\ &= \frac{1}{2} \left[ \log \frac{s^2+a^2}{s^2+b^2} \right]_s^{\infty} \\ &= \frac{1}{2} \left[ 0 - \log \frac{s^2+a^2}{s^2+b^2} \right] \\ &= -\frac{1}{2} \log \left( \frac{s^2+a^2}{s^2+b^2} \right) \end{aligned}$$

5) Find the Laplace transform of  $e^{-t} \int_0^t t \cos t dt$

Sol:  $L \left[ e^{-t} \int_0^t t \cos t dt \right] = \left[ L \left( \int_0^t t \cos t dt \right) \right]_{s \rightarrow s+1}$

$\left( \because L \int_0^t f(t) dt = \frac{1}{s} L[f(t)] \right)$

$$\begin{aligned} &= \left[ \frac{1}{s} L(t \cos t) \right]_{s \rightarrow s+1} \\ &= \left[ \frac{1}{s} \left( -\frac{d}{ds} L(\cos t) \right) \right]_{s \rightarrow s+1} \\ &= \left[ \frac{-1}{s} \frac{d}{ds} \left( \frac{s}{s^2+1} \right) \right]_{s \rightarrow s+1} \\ &= \cancel{\left( \frac{s^2}{s^2+1} \right)} = \left[ -\frac{1}{s} \left( \frac{s^2+1-2s^2}{(s^2+1)^2} \right) \right]_{s \rightarrow s+1} \\ &= \left[ \frac{-1}{s} \left( \frac{1-s^2}{(s^2+1)^2} \right) \right]_{s \rightarrow s+1} \end{aligned}$$



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$$= \left[ \frac{s^2 - 1}{s(s^2 + 1)^2} \right]_{s \rightarrow s+1}$$

$$= \frac{(s+1)^2 - 1}{(s+1)((s+1)^2 + 1)^2}$$

$$= \frac{s^2 + 2s}{(s+1)(s^2 + 2s + 2)^2}$$

$$L \left[ e^{-t} \int_0^t t \cos t \, dt \right] = \frac{s^2 + 2s}{(s+1)(s^2 + 2s + 2)^2}$$

6) Evaluate using Laplace transform  $\int_0^\infty t e^{-2t} \sin 3t \, dt$

$$\text{sol: } \int_0^\infty t e^{-2t} \sin 3t \, dt = \int_0^\infty e^{-2t} (t \sin 3t) \, dt$$

$$= \left[ \int_0^\infty e^{-st} (t \sin 3t) \, dt \right]_{s=2}$$

$$= [L(t \sin 3t)]_{s=2}$$

$$= \left[ -\frac{d}{ds} L(\sin 3t) \right]_{s=2}$$

$$= \left( -\frac{d}{ds} \left( \frac{3}{s^2 + 9} \right) \right)_{s=2}$$

$$= \left( \frac{6s}{(s^2 + 9)^2} \right)_{s=2}$$

$$= \frac{12}{169}$$