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UNIT 4 Fourier Series and Fourier Transform

PROBLEMS UNDER FOURIER SERIES IN (0.201).
D Hind the Fourier Series of
$$f(x) = \frac{\pi}{2}$$
;
O $2x + 22\pi$. Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \cdots = \frac{\pi}{4}$.
Solution:
 $f(x)$ is defined in $(0, 2\pi)$.
Ho doublier sources for $f(x)$ in the interval
 $(0, 2\pi)$ is given by
 $f(x) = \frac{\alpha_0}{2} + \bigotimes_{n=1}^{\infty} \alpha_n \cos nx + \bigotimes_{n=1}^{\infty} b_n \sin nx = 0$
 $\alpha_0 = \frac{1}{2\pi} \int_{1-x}^{2\pi} f(x) dx$
 $= \frac{1}{2\pi} \int_{1-x}^{2\pi} (\pi - x) dx$
 $= \frac{1}{2\pi} \left[\pi - \frac{2\pi}{2} \right]_{0}^{2\pi}$
 $= \frac{1}{2\pi} \left[2\pi^2 - \frac{(2\pi)^2}{2} - 0 - 0 \right]$
 $= \frac{1}{2\pi} \left[2\pi^2 - \frac{4\pi^2}{2} \right] = \frac{1}{2\pi} (0)$
 $(\alpha = 0)$

23MAT103-Differential Equations and Transforms





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$a_n = \frac{1}{JI} \int f(x) \cos nx dx$
$= \frac{1}{2\pi} \int \frac{\sqrt{1-x}}{\sqrt{2}} \cos nx dx$
$= \frac{1}{2\pi} \left[(\pi - 2) \left(\frac{\sin nx}{n} \right) - (0 - 1) \left(-\frac{\cos nx}{n^2} \right)^2 \right]$
$=\frac{-1}{2\pi n^2} \left[\cos n \Delta \right]_0^{2\pi}$
$= \frac{-1}{2\pi n^2} \left(\sum \cos 2n\pi - \cos \alpha \right)$
$= \frac{-1}{2\pi n^2} [1 - 1] = \frac{-1}{2\pi n^2} (0)$
Q = D
$bn = \frac{1}{21} \int f(x) \sin nx dx$
$= \frac{1}{T} \int_{0}^{2\pi} \left(\frac{\pi - x}{2}\right) \sin nx dx$
$xb xn nik (x-\pi C) = \frac{\pi c}{2}$

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Mr. C. Saminathan /AP/MATHS





$$= \frac{1}{2\pi} \left[(J_{-} x) \left(\frac{\cos nx}{n} \right) - (0 - i) \left(\frac{\sin nx}{n} \right) \right]_{0}^{2T}$$

$$= \frac{1}{2\pi} \left[(J_{-} x) \cos nx \right]_{0}^{2T}$$

$$= \frac{1}{2\pi\pi} \left[(J_{-} x) \cos nx \right]_{0}^{2T}$$

$$= \frac{-1}{2\pi\pi} \left[(J_{-} x) \cos nx \right]_{0}^{2T}$$

$$= \frac{-1}{2\pi\pi} \left[(J_{-} x) \cos nx \right]_{0}^{2T}$$

$$= \frac{-1}{2\pi\pi} \left[(J_{-} x) \cos nx - (J_{-} x) \cos 0 \right]$$

$$= \frac{-1}{2\pi\pi} \left[(J_{-} x) (I_{-} - T_{-} x) - (J_{-} x) \cos 0 \right]$$

$$= \frac{-1}{2\pi\pi} \left[(J_{-} x) (I_{-} - T_{-} x) - (J_{-} x) - (J_{-} x) \right]$$

$$= \frac{1}{2\pi\pi} \left[(J_{-} x) (I_{-} - T_{-} x) - (J_{-} x) - (J_{-} x) \right]$$

$$= \frac{1}{2\pi\pi} \left[(J_{-} x) (I_{-} x) - (J_{-} x) - (J_{-} x) - (J_{-} x) - (J_{-} x) \right]$$

$$= \frac{1}{2\pi\pi} \left[(J_{-} x) (I_{-} x) - (J_{-} x) \right]$$

$$= \frac{1}{2\pi\pi} \left[(J_{-} x) (I_{-} x) (I_{-} x) - (J_{-} x) - (J_{-} x) - (J_{-} x) - (J_{-} x) \right]$$

$$= \frac{1}{2\pi\pi} \left[(J_{-} x) (I_{-} x) (I_{-} x) - (J_{-} x) - (J_{-} x) - (J_{-} x) - (J_{-} x) \right]$$

$$= \frac{1}{2\pi\pi} \left[(J_{-} x) (I_{-} x) (I_{-} x) - (J_{-} x) - (J_{-} x) - (J_{-} x) - (J_{-} x) \right]$$

$$= \frac{1}{2\pi\pi} \left[(J_{-} x) (I_{-} x) (I_{-} x) - (J_{-} x) - (J_{-} x) - (J_{-} x) \right]$$

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$$= \frac{1}{2\pi\pi} \left[(J_{-} x) (I_{-} x) (I_{-} x) (I_{-} x) (I_{-} x) (I_{-} x) - (J_{-} x) - (J_{-} x) \right]$$

$$= \frac{1}{2\pi\pi} \left[(J_{-} x) (I_{-} x) (I$$



Here
$$x = \prod_{1}^{n}$$
 lies inside $(0, 2\pi)$. Hence $f(x)$
unitinues at $x = \frac{\pi}{2}$
 $f\left(\frac{\pi}{2}\right) = \frac{(1-\pi)/2}{2} = \frac{\pi l_2}{2} = \frac{\pi}{4}$
Substitute the value of $f\left(\frac{\pi}{2}\right)$ in (3), we get
 $\frac{\pi}{4} = \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{2n\pi}{2}$
 $\therefore \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{2\pi}{2} = \frac{\pi}{4}$
 $\sin \frac{\pi}{2} + \frac{1}{2} \sin \frac{2\pi}{2} + \frac{1}{3} \sin \frac{3\pi}{2} + \dots = \frac{\pi}{4}$
 $1 + D + \frac{1}{3}(-1) + \dots = \frac{\pi}{4}$
 $1 + D + \frac{1}{3}(-1) + \dots = \frac{\pi}{4}$
Ans: $\left[\frac{1-\frac{1}{3}}{\frac{1}{1}} + \frac{1}{5} - \dots = \frac{\pi}{4}\right]$
 $i)\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{5}$
 $ii)\frac{1}{1^2} + -\frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{5}$
 $iii)\frac{1}{4^2} + \frac{1}{3^2} + \frac{1}{5^2} - \dots = \frac{\pi^2}{5}$







$$\underbrace{\underbrace{Soln:}_{0}}_{q_{1}} f(x) \text{ is defined in } (0, 2\pi)}_{q_{1}}$$

$$\underbrace{\operatorname{He}}_{1 \text{ fourtion survives for } f(x) + f(x) \text{ in the sin } (0, 2\pi)}_{1 \text{ interval } (0, 2\pi)}_{q_{2}} \text{ is given by,}$$

$$f(x) = \frac{a_{0}}{2} + \underbrace{\sum_{n=1}^{2} a_{n} \cos nx + \sum_{n=1}^{2} b_{n} \sin nx}_{n=1} = 0$$

$$a_{0} = \frac{1}{\pi} \int_{0}^{2\pi} f(x) \, dx$$

$$= \frac{1}{\pi} \left[2\pi \frac{x_{2}}{2} - \frac{x^{2}}{3} \right]_{0}^{2\pi}$$

$$= \frac{1}{\pi} \left[\pi (2\pi)^{2} - \frac{(2\pi)^{3}}{3} - 0 - 0 \right]$$

$$= \frac{1}{\pi} \left[\frac{12\pi^{3} - 8\pi^{3}}{3} \right]$$

$$= \frac{1}{\pi} \left[\frac{12\pi^{3} - 8\pi^{3}}{3} \right]$$

$$a_{0} = \frac{4\pi^{2}}{3}$$

$$a_{0} = \frac{4\pi^{2}}{3}$$

$$a_{0} = \frac{1}{\pi} \int_{0}^{2\pi} f(x) \cos nx \, dx$$

$$= \frac{1}{\pi} \int_{0}^{2\pi} (2\pi x - x^{2}) \cos nx \, dx$$





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$$= \frac{1}{3\pi} \int \frac{(2372 - 22)}{(237 - 22)} \left(\frac{\sin 62}{n} - (237 - 22) \left(\frac{\cos nx}{n^2} \right) + (6 - 2) \left(\frac{1}{n^2} - \frac{1}{3} \right) \left[(237 - 437) - (237 - 1) - (237 - 1) - \frac{1}{n^2} \right]$$

$$= \frac{1}{3\pi} \int \frac{(-237 - 437)}{(-27 - 1)^2} - \frac{1}{n^2} - \frac{1}{n^2}$$

$$= \frac{1}{3\pi} \int \frac{(-437)}{(-437)}$$

$$= \frac{1}{3\pi} \int \frac{-437}{(-27)^2} + \frac{1}{(-27)^2} + \frac{1}{$$

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$$i = \frac{1}{J_{1}} \left[\frac{-2}{n^{3}} + \frac{2}{n^{3}} \right]$$

$$\boxed{bn = 0}$$
Substitute the values of a_{0} , a_{1} , b_{1} in (1), we get
$$f(x) = \frac{9J_{1}^{2}}{3} - 4 \sum_{n=1}^{\infty} \frac{1}{n^{2}} \cos nx - 2$$
i) To deduce $\frac{1}{1^{2}} + \frac{1}{2^{2}} + \dots = \frac{J_{1}^{2}}{6}$
Rut $x = 0$ in (2), we get
$$f(0) = \frac{9J_{1}^{2}}{3} - 4 \sum_{n=1}^{\infty} \frac{1}{n^{2}} \cos 0$$

$$= \frac{9J_{1}^{2}}{3} - 4 \sum_{n=1}^{\infty} \frac{1}{n^{2}} - 3$$
To find $f(0)$:
(We have $f(x) = 2J(x - x^{2}), 0 < x < 2J$

$$f(0) = 0$$

$$f(0) = 2J(2J_{1}) - (2J_{1})^{2}$$

$$= 4J^{2} - 4J^{2}$$

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 $f(0) = f(0\pi) = 0$ Ms.C.Saranya, AP/Maths





Substitute the value of f(0) is 3 we get
$b = \frac{2\pi^2}{3} - 4 \sum_{n=1}^{\infty} \frac{1}{n^2}$
$4 \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{2\pi^2}{3}$
$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$
$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{5} - \frac{\pi}{6}$
ii) To deduce $\frac{1}{1^2} = \frac{1}{2^2} = \frac{1}{3^2} = \frac{\pi^2}{12}$
Put $2=31$ in (a) we get $f(51) = \frac{251^2}{3} - 4 \sum_{n=1}^{\infty} \frac{1}{n^2} \cos n57$ in (b)
$f(\pi) = \frac{2\pi^2}{3} - 4 \stackrel{2}{\leq} \frac{1}{n^2} (-0)^n - 4$
To find f(IT):
2c=JT lies Inside (0,2J) ∴ for is continuous at 2=J
$f(\pi) = 2\pi^2 - \pi^2$ $f(\pi) = \pi^2$
Substitute the value of f(J) in (4), we get





$$\pi^{2} = \frac{2\pi^{2}}{3} - 4 \sum_{n=1}^{\infty} \frac{(-1)^{2}}{n^{2}}$$

$$-4 \sum_{n=1}^{\infty} \frac{(-1)^{n}}{n^{2}} = \pi^{2} - \frac{2\pi^{2}}{3}$$

$$= \frac{3\pi^{2} - 2\pi^{2}}{3}$$

$$= \frac{3\pi^{2} - 2\pi^{2}}{3}$$

$$= \frac{3\pi^{2}}{3}$$

$$= \frac{\pi^{2}}{3}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n}}{n^{2}} = -\frac{-\pi^{2}}{12}$$

$$-\frac{1}{1^{2}} + \frac{1}{2^{2}} - \frac{1}{3^{2}} + \dots = -\frac{\pi^{2}}{12}$$

$$\frac{1}{1^{2}} - \frac{1}{2^{2}} + \frac{1}{3^{2}} - \dots = \frac{\pi^{2}}{12}$$

$$\frac{1}{1^{2}} - \frac{1}{2^{2}} + \frac{1}{3^{2}} - \dots = \frac{\pi^{2}}{12}$$

$$\frac{1}{1^{2}} - \frac{1}{2^{2}} + \frac{1}{3^{2}} + \dots = \frac{\pi^{2}}{8}$$
Adding (a) and (b), we get
$$An_{0}: \left[\frac{1}{1^{2}} + \frac{1}{3^{2}} + \frac{1}{5^{2}} + \dots = \frac{\pi^{2}}{8}\right]$$





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Homework sums:
(1) Find the Fourier series for
$$f(x) = x$$
 is
 $0 \le x \le 22\pi$. Hence dedute $1 - \frac{1}{3} + \frac{1}{5} - \dots = \frac{\pi}{4}$
Ans: $f(x) = \pi - 2 \stackrel{20}{>} \frac{1}{5} \sin nx$
 $n = 1$
(2) Find the Fourier series of $f(x) = (\pi - x)^2$
in $(0, 2\pi)$ by periodicity 2π
Ans: $f(x) = \frac{\pi^2}{3} + 4 \begin{bmatrix} \cos x + \cos 2x + \cos 3x + \dots \\ 1^2 + 2^2 + 3^2 + \dots \end{bmatrix}$
(3) Find the Fourier series for $f(x) = x^2$;
 $0 \le 2x \le 2\pi$. Hence dedute
 $a) = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{12}$
 $b) = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{12}$
 $c) = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{12}$
Ans: $f(x) = \frac{4\pi^2}{3} + 4 \stackrel{20}{>} \frac{1}{1^2} \cos nx$
 $n = 1$
 $m_{x=1}$ Mr. C Saminathan /AP/MATHS

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