



## ODD AND EVEN FUNCTIONS:

### EVEN FUNCTION:

Let  $f(x)$  be defined in  $(-l, l)$

If  $f(-x) = f(x)$ , then  $f(x)$  is an even function.

#### Note:

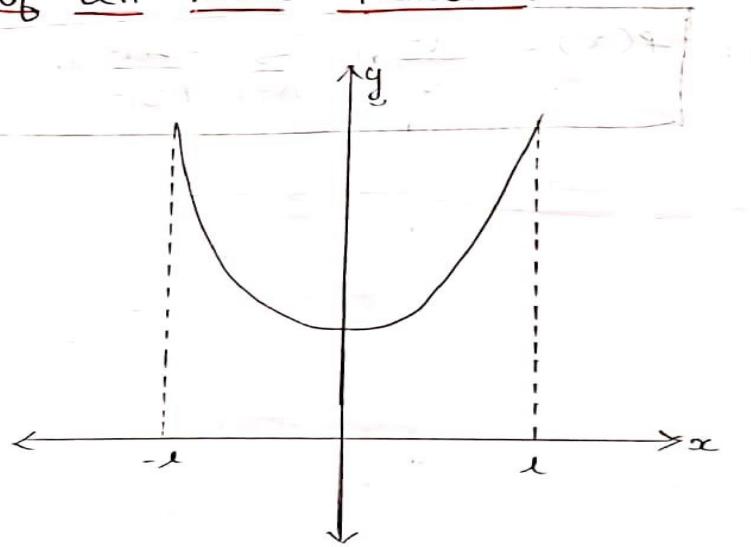
1. The graph of even function is symmetrical about y-axis

2.  $\int_{-l}^l f(x) dx = 2 \int_0^l f(x) dx$  if  $f(x)$  is even.

3. Sum of two even functions is also an even function.

4. Product of two even functions is also an even function.

### Graph of an Even Function:





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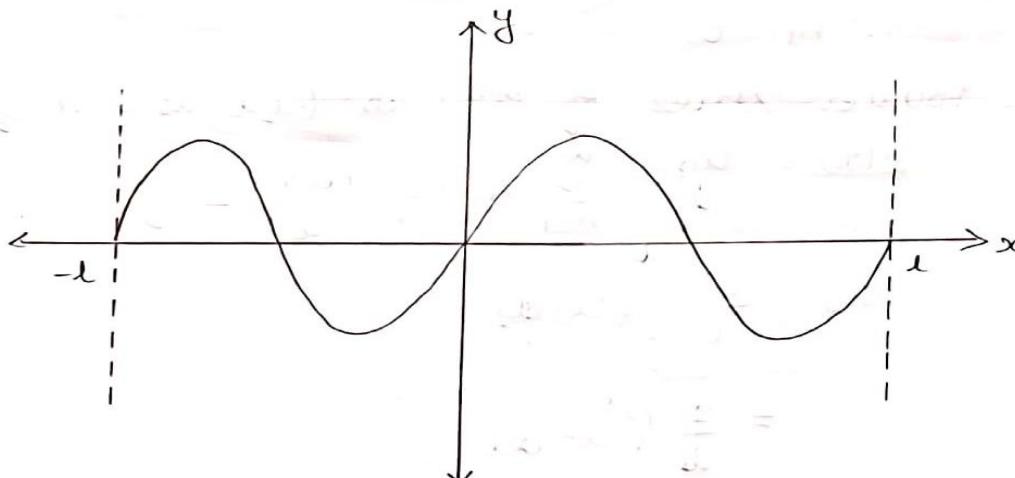
### ODD FUNCTION:

Let  $f(x)$  be defined in  $(-l, l)$ :  
If  $f(-x) = -f(x)$ , then  $f(x)$  is an odd function.

#### Note:

1. The graph of odd function is symmetrical about origin.
2.  $\int_{-l}^l f(x) dx = 0$  if  $f(x)$  is odd
3. Sum of two odd function is also an odd function.
4. Product of two odd function is an even function.

### Graph of an Odd Function:





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- ③ Find the Fourier series for  $f(x) = x$  in  $(-l, l)$  and hence deduce that

$$1 - \frac{1}{3} + \frac{1}{5} - \dots = \frac{\pi}{4}$$

Solution:

$f(x)$  is defined in  $(-l, l)$ .  
In  $(-l, l)$ , check whether  $f(x)$  is odd / even.

$$f(x) = x$$

$$f(-x) = -x = -f(x)$$

$$\therefore f(-x) = -f(x)$$

$$\therefore a_0 = a_n = 0$$

The Fourier series is given by

For  $f(x)$  is  $(-l, l)$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \quad \text{--- (1)}$$

$$b_n = \frac{2}{l} \int_0^l f(x) \sin \frac{n\pi x}{l} dx$$

$$= \frac{2}{l} \int_0^l x \sin \frac{n\pi x}{l} dx$$

$$= \frac{2}{l} \left[ x \left( \frac{-\cos n\pi x}{l} \right) \Big|_0^l - \left( \frac{-\sin n\pi x}{l} \right) \Big|_0^l \right]$$

$$= -\frac{2}{l} \cdot \frac{l}{n\pi} \left[ x \cos \frac{n\pi x}{l} \Big|_0^l \right]$$



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$$= \frac{-2}{n\pi} [l \cos n\pi - 0]$$

$$\boxed{b_n = \frac{-2l}{n\pi} (-1)^n}$$

Substitute the value of  $b_n$  in ①,  
we get

$$f(x) = \sum_{n=1}^{\infty} \frac{-2l}{n\pi} (-1)^n \sin \frac{n\pi x}{l} \quad \text{--- } ②$$

To deduce  $1 - \frac{1}{3} + \frac{1}{5} - \dots = \frac{\pi}{4}$ .

Put  $x = \frac{l}{2}$  in ② we get

$$f\left(\frac{l}{2}\right) = \sum_{n=1}^{\infty} \frac{-2l}{n\pi} (-1)^n \sin \frac{n\pi}{2} \quad \text{--- } ③$$

To find  $f\left(\frac{l}{2}\right)$ :

$x = \frac{l}{2}$  lies inside  $(-l, l)$ .

Here  $x = \frac{l}{2}$  is a point of continuity.

$$f\left(\frac{l}{2}\right) = \frac{l}{2}.$$

$$③ \Rightarrow \frac{l}{2} = \sum_{n=1}^{\infty} \frac{-2l}{n\pi} (-1)^n \sin \frac{n\pi}{2}.$$

$$-\frac{2l}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi}{2} = \frac{l}{2}.$$



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$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi}{2} = \frac{1}{2} \left( \frac{\pi}{-2} \right)$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi}{2} = -\frac{\pi}{4}$$

$$\frac{(-1)}{1}(1) + 0 + \frac{(-1)}{3}(-1) + \dots = -\frac{\pi}{4}$$

Ans : 
$$\boxed{1 - \frac{1}{3} + \frac{1}{5} - \dots = \frac{\pi}{4}}$$

### Homework sums:

- ① Find the fourier series for  $f(x) = x^2$  in  $(-\pi, \pi)$  and hence deduce

$$i) \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6} \quad ii) \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$$

Ans: 
$$\boxed{f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2} (-1)^n \cos nx}$$

- ② Find the Fourier series for

$$f(x) = \begin{cases} 1 + \frac{x}{\pi} & ; -\pi < x \leq 0 \\ 1 - \frac{x}{\pi} & ; 0 < x < \pi \end{cases}$$

and hence

deduce  $\frac{1}{1^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{8}$

Ans: 
$$\boxed{f(x) = \frac{1}{2} + \sum_{n=1,3,5}^{\infty} \frac{4}{n^2 \pi^2} \cos nx}$$