19MEB303– FINITE ELEMENT ANALYSIS

IAE 3 FEA- Answer Key

SET A

PART A – Short Answers

1. Finite element equation for 1D heat conduction with free end convection:

$$[K]\{T\} = \{F\}$$

where

$$K = \int_0^L kA\frac{dN^T}{dx}\frac{dN}{dx}dx + \int_0^L hPN^TNdx \quad F = \int_0^L hPN^TT_\infty dx$$

2. Conduction, convection, and thermal load matrices for 1D heat transfer through a fin:

- Conduction:

$$[K_c] = \int_0^L kA\frac{dN^T}{dx}\frac{dN}{dx}dx$$

- Convection:

$$[K_h] = \int_0^L hPN^TNdx$$

- Thermal load:

$$\{F\} = \int_0^L hPN^TT_\infty dx$$

3. Heat loss from the pipe:

Given:

- Diameter $D = 0.1$ m
- $T_s = 250^\circ C, T_\infty = 20^\circ C$
- h = 20 \, \text{W/m}^2^\circ C

$$Q = hA(T_s - T_\infty) = 20 \cdot \pi \cdot 0.1 \cdot (250 - 20) = \boxed{1445.13\,\text{W/m}}$$

4. Difference between element types:
    - **Subparametric:** Geometry shape functions < displacement shape functions
    - **Isoparametric:** Geometry and displacement shape functions are same
    - **Superparametric:** Geometry shape functions > displacement shape functions
5. Gaussian quadrature expression:

$$\int_{-1}^{1} f(x)dx \approx \sum_{i=1}^{n} w_i f(x_i)$$

   where x_i are Gauss points and w_i are weights

Part B

▢ Q6 (a) – Composite Wall Temperature Distribution

Steps:

1. Convert thickness to meters.
2. Calculate thermal resistances for each layer and convection.
3. Compute total thermal resistance.
4. Find heat transfer rate:

$$q = \frac{T_{\text{inside}} - T_\infty}{R_{\text{total}}}$$

5. Calculate interface temperatures using $T = T_1 - q \cdot R$

Final Answers:

- $q = 60.15 \, \text{W/m}^2$
- $T_2 = 189.85°C, T_3 = 39.47°C, T_4 = 30.77$

□ Q6 (b) – Fin Temperature Distribution

Steps:

1. Compute cross-sectional area and perimeter.
2. Calculate $m = \sqrt{\frac{hP}{kA}}$
3. Use the 1D fin formula with insulated sides:

$$T(x) = T_\infty + (T_b - T_\infty) \cdot \frac{\cosh(m(L - x))}{\cosh(mL)}$$

Final Expression:

$$T(x) = 30 + 300 \cdot \frac{\cosh(0.1266(120 - x))}{\cosh(15.19)}$$

□ Q7 (a) – Strain-Displacement Matrix for 4-Node Quad

Steps:

1. Write shape functions N_1 to N_4.
2. Use chain rule with Jacobian to convert $\frac{\partial N}{\partial \xi}, \eta \rightarrow x, y$
3. Form B-matrix with partial derivatives:

$$B = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \end{bmatrix}$$

Final Answer:

Strain-displacement matrix $[B]$ at Gauss point using Jacobian.

□ Q7 (b) (i) – Gauss Integration of Polynomial

Steps:

1. Use 2-point Gauss quadrature with points $\pm 1/\sqrt{3}$ and weights = 1.
2. Evaluate the function at both points.

Final Answer:

Exact value and Gauss approximation = 4

□ Q7 (b) (ii) – 3-Point Gaussian Quadrature

Steps:

1. Use standard 3-point rule:
   - Points: $\pm\sqrt{3/5}, 0$
   - Weights: $\frac{5}{9}, \frac{8}{9}, \frac{5}{9}$
2. Evaluate function at each point and apply weights.

Final Answer:

Numerical value based on specific function.

□ Q8 (a) – 1D Finite Element with Convection

Steps:

1. Write element stiffness matrix including conduction and convection:

$$K_e = \quad (kA\frac{dN^T}{dx}\frac{dN}{dx} + hPN^T N)dx$$

2. Form load vector from convection.

Final Answer:

Stiffness matrix and load vector with k, A, h, P, L, T_∞

□ Q8 (b) – Composite Wall 2

Steps:

1. Same as Q6(a) with different dimensions and conductivities.
2. Calculate resistances, total resistance, heat transfer, and interface temperatures.

Final Answer:

- Heat loss: $q \approx 59.18\,W$
- Interface temperatures:
  - $T_2 \approx 162.01^\circ C$
  - $T_3 \approx 42.02^\circ C$
  - $T_4 \approx 33.79^\circ C$