

## DEPARTMENT OF MATHEMATICS

Course Code & Name: **23MAT101 -MATRICES AND CALCULUS**

Course Faculty : **MAREESWARL.R**

### Question Bank

#### UNIT I-MATRIX EIGENVALUE PROBLEMS

#### Part A-Two MarkS Questions

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|---|-----------|------|
| 1. If 1, 2, -1 are the eigen values of the matrix A, Estimate the eigen values of the matrix $A^3$  | GATE-2016 | L -2 |
| 2. Can $\begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix}$ is diagonalizable. Why?  | GATE-2016 | L -2 |
| 3. Write the matrix form of the quadratic form<br>$6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$  | GATE-2020 | L -2 |
| 4. Find the nature of the quadratic form whose canonical form is<br>$y_1^2 + y_2^2 + 4y_3^2$  | GATE-2016 | L -2 |
| 5. Find the curvature at any point on the curve $2x^2 + 2y^2 - 5x - 2y + 1 = 0$   | GATE-2020 | L -2 |
| 6. Determine the nature of the quadratic form $x_1^2 + 2x_2^2$  | GATE-2022 | L -2 |
| 7. Define Evolute and involute  | GATE-2016 | L -2 |
| 8. Find the envelope of $y = mx - \sqrt{1 + m^2}$ .   | GATE-2018 | L-2  |
| 9. Solve the equation $(D^2 - 2D + 2)y = 0$   | GATE-2020 | L-3  |
| 10. If the eigen values of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ are 0, 3, 15, find the eigen vectors |           | L-2  |

### Part B-16 Mark Questions

1. Find the eigen values and eigen vectors of the matrix	$\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$	GATE-2017	L-3
2. Verify Cayley Hamilton Theorem for the matrix $A^4$	$A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ . Also deduce the	GATE-2016	L-3
3. Find a change of variables that reduces the quadratic form $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ to a sum of squares and express the quadratic Form in terms of new variables.		GATE-2020	L-2
4. Determine the Evolute of the Parabola $y^2 = 4ax$		GATE-2018	L-2
5. Find the radius of curvature of $xy = c^2$ at $(c, c)$			L-2
6. Find the centre and circle of curvature $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point $(\frac{a}{4}, \frac{a}{4})$		GATE-2020	L-2
7. Find the evolute of Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$		GATE-2021	L-2
8. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$ , find its $A^{-1}$ .		GATE-2016	L-3
9. Prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$ using Euler's theorem if $u = \sin^{-1} \left( \frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}} \right)$ .		GATE-2017	L-3
10. Find the envelope of $\frac{x}{a} + \frac{y}{b} = 1$ , where $a$ and $b$ are parameters subject to (i) $a + b = c$ , (ii) $ab = c^2$ , $c$ is a constant.		GATE-2019	L-2
11. Find the radius of curvature at the point $(0, c)$ on the curve $y = c \cosh \frac{x}{c}$ .		GATE-2018	L-2
12. Find the evolute of the cycloid $x = a(\theta - \sin \theta)$ , $y = a(1 - \cos \theta)$ .		GATE-2017	L-3

13. If $y = \frac{ax}{a+x}$ , prove that $\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$ , where $\rho$ is the radius of curvature.	GATE-2021	L-3
14. Show that the evolutes of the parabola $y^2 = 4ax$ is the curve $27ay^2 = 4(x-2a)^3$ .	GATE-2019	L-3
15. Prove that the radius of the curve $xy^2 = a^3 - x^3$ at the point $(a,0)$ is $\frac{3a}{2}$ .	GATE-2018	L-3
16. Find the radius of curvature $\sqrt{x} + \sqrt{y} = 1$ at the point $\left(\frac{1}{4}, \frac{1}{4}\right)$ .	GATE-2020	L-2