

THEORY COURSE

ENGINEERING MECHANICS

Simple Harmonic Motions

23ASE306 Theory of Vibrations and Aeroelasticity



INSTRUCTOR

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COURSE CODE

23ASE306

Aerospace Engineering

COURSE GOALS

Learning Objectives

By the end of this session, students will be able to:

✓ **Define SHM**
Understand periodic motion with proportional restoring force

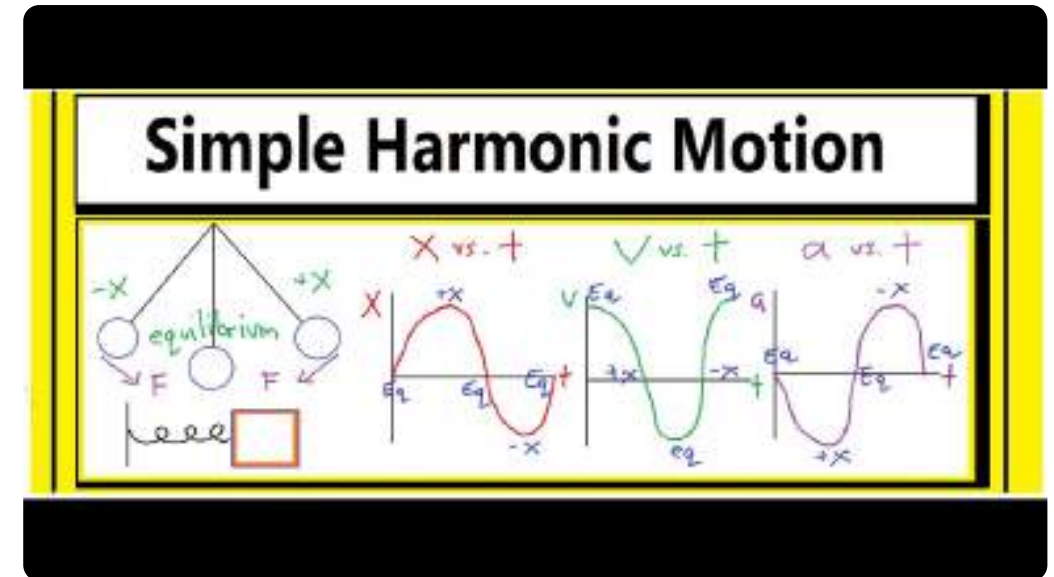
√x **Mathematical Model**
Apply equations for displacement, velocity, and acceleration

⚙️ **Mass-Spring System**
Analyze classic SHM example with natural frequency

⚡ **Energy Conservation**
Understand kinetic and potential energy transformation

🕒 **Phase & Frequency**
Apply relationships between period, frequency, and phase angle

SHM Motion Characteristics



↕ Displacement

⦿ Velocity

⦿ Acceleration

Recap: Mechanical Vibration Terminologies

Key concepts from previous session

Basic Motion Terms

x - Displacement (m)

v - Velocity (m/s)

a - Acceleration (m/s²)

Wave Parameters

A - Amplitude

f - Frequency (Hz)

T - Period (s)

φ - Phase angle

System Characteristics

ω_n - Natural frequency

ζ - Damping ratio

Resonance phenomena

Measurement Terminology

RMS - Effective value

Peak - Maximum value

Accelerometer sensor

Frequency spectrum analysis

Key Relationships

Frequency & Period

$$T = 1/f$$

Angular Frequency

$$\omega = 2\pi f$$

Natural Frequency

$$\omega_n = \sqrt{k/m}$$

What is Simple Harmonic Motion?

Fundamentals of periodic vibratory motion



Definition

CORE CONCEPT

Periodic motion where the **restoring force** is **directly proportional** to displacement and acts in the **opposite direction**.

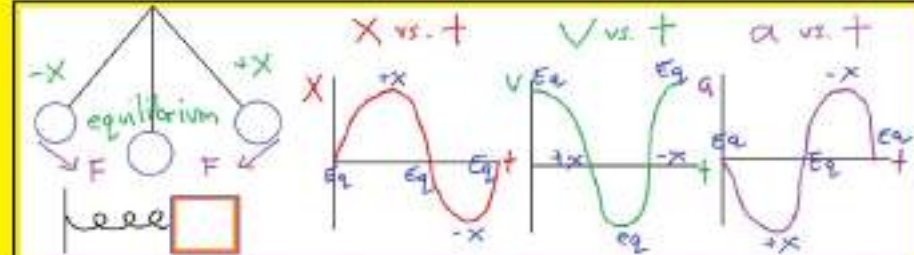
$$F = -kx$$

★ Key Characteristics

- 1 **Sinusoidal Motion**
Displacement varies sinusoidally with time
- 2 **Constant Period**
Time for one complete cycle remains constant
- 3 **No Energy Loss**
Ideal system with zero damping ($\zeta = 0$)

SHM Motion Patterns

Simple Harmonic Motion



DISPLACEMENT

$$x = A \cos(\omega t)$$

VELOCITY

$$v = -A\omega \sin(\omega t)$$

ACCELERATION

$$a = -A\omega^2 \cos(\omega t)$$

Mathematical Representation of SHM

Equations describing displacement, velocity, and acceleration

↑ **Displacement**
Position vs Time

$$x = A \cos(\omega t + \phi)$$

● Reference

$\phi = 0$ at $t = 0$

🕒 **Velocity**
Rate of Change

$$v = -A\omega \sin(\omega t + \phi)$$

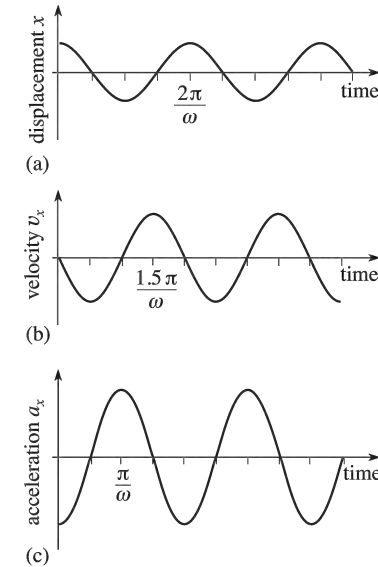
● +90° Phase

Leads displacement

🚀 **Acceleration**
Second Derivative

$$a = -A\omega^2 \cos(\omega t + \phi)$$

Phase Relationships



x
Reference
0° phase

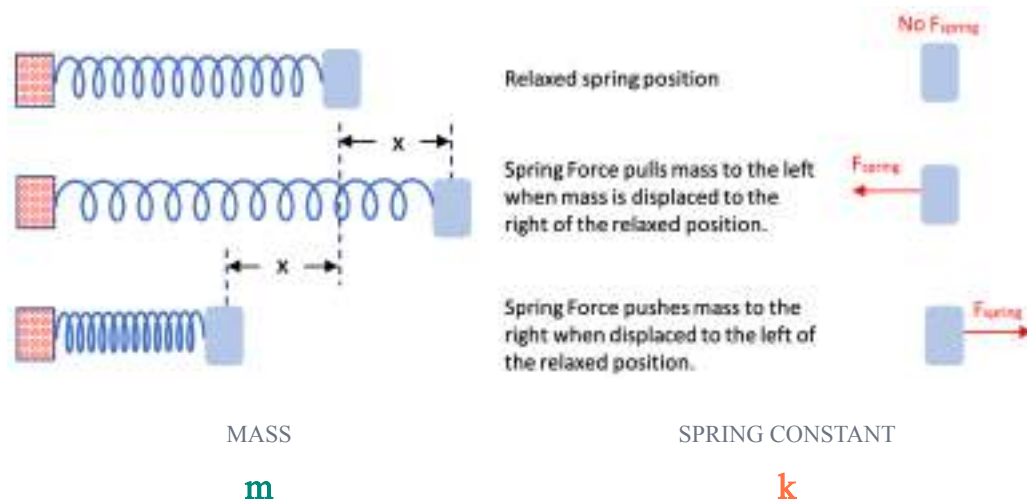
v
Derivative
+90° phase

a
2nd Derivative
+180° phase

Mass-Spring System: Classic SHM Example

Fundamental model for vibratory systems

System Configuration



Hooke's Law

Restoring Force

$$F = -kx$$

Force proportional to displacement



Natural Frequency

System's Inherent Frequency

$$\omega_n = \sqrt{k/m}$$

Angular frequency

rad/s



System Parameters

Stiffness (k): N/m

Mass (m): kg

Damping: Zero (ideal)



Equation of Motion

Differential Equation

$$m\ddot{x} + kx = 0$$

Energy in Simple Harmonic Motion

Energy conservation and transformation in vibratory systems

 **Total Energy**
CONSTANT SYSTEM

$$E = \frac{1}{2}kA^2$$

Remains constant throughout motion

 **Kinetic Energy**
Motion Energy

$$KE = \frac{1}{2}mv^2$$

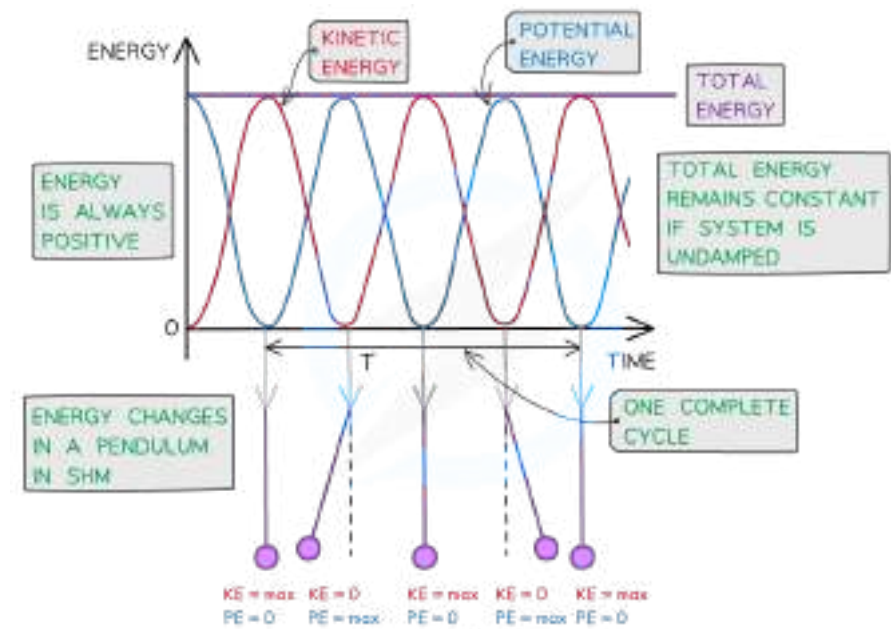
Maximum at equilibrium

$$v = v_{\max}$$

 **Potential Energy**
Stored Energy

$$PE = \frac{1}{2}kx^2$$

↔ Energy Transformation



KE

Max at $x = 0$
Zero at $x = \pm A$

PE

Max at $x = \pm A$
Zero at $x = 0$

∞ **Energy Conservation:** $KE + PE = \text{Constant}$

Phase and Frequency in SHM

Temporal and phase relationships in harmonic motion



Phase Angle (ϕ)

Initial Position

$$\phi = \omega t_0 + \phi_0$$

$$\phi = 0$$

Start at A

$$\phi = \pi/2$$

Start at 0

$$\phi = \pi$$

Start at -A



Frequency (f)

Cycles per Second

$$f = \omega/2\pi$$

Units:

Hertz (Hz)



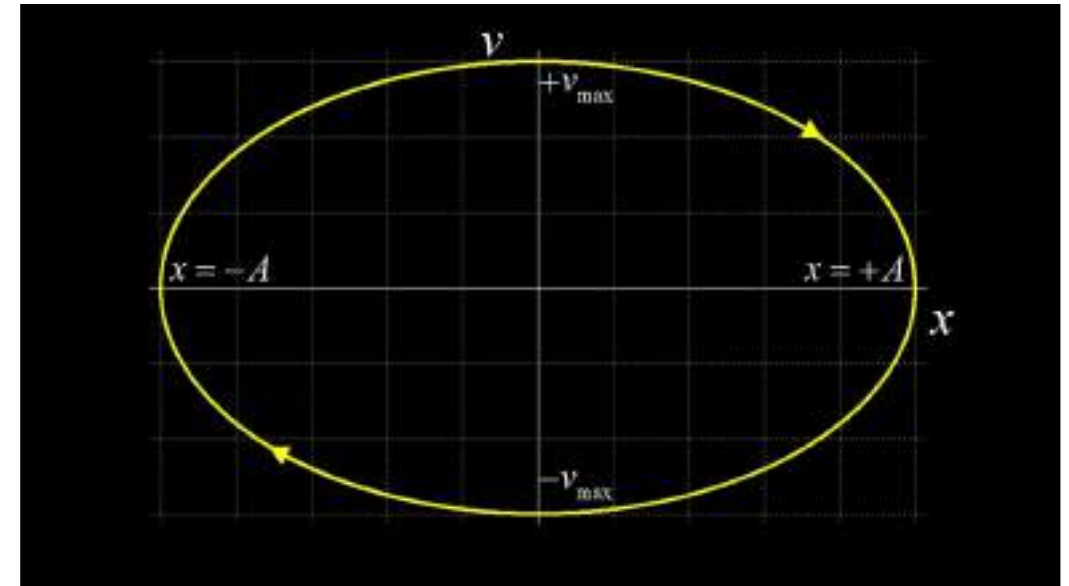
Period (T)

Time for One Cycle

$$T = 2\pi/\omega$$



Phase Space Trajectory



TRAJECTORY

Closed Ellipse

Periodic Motion

AREA

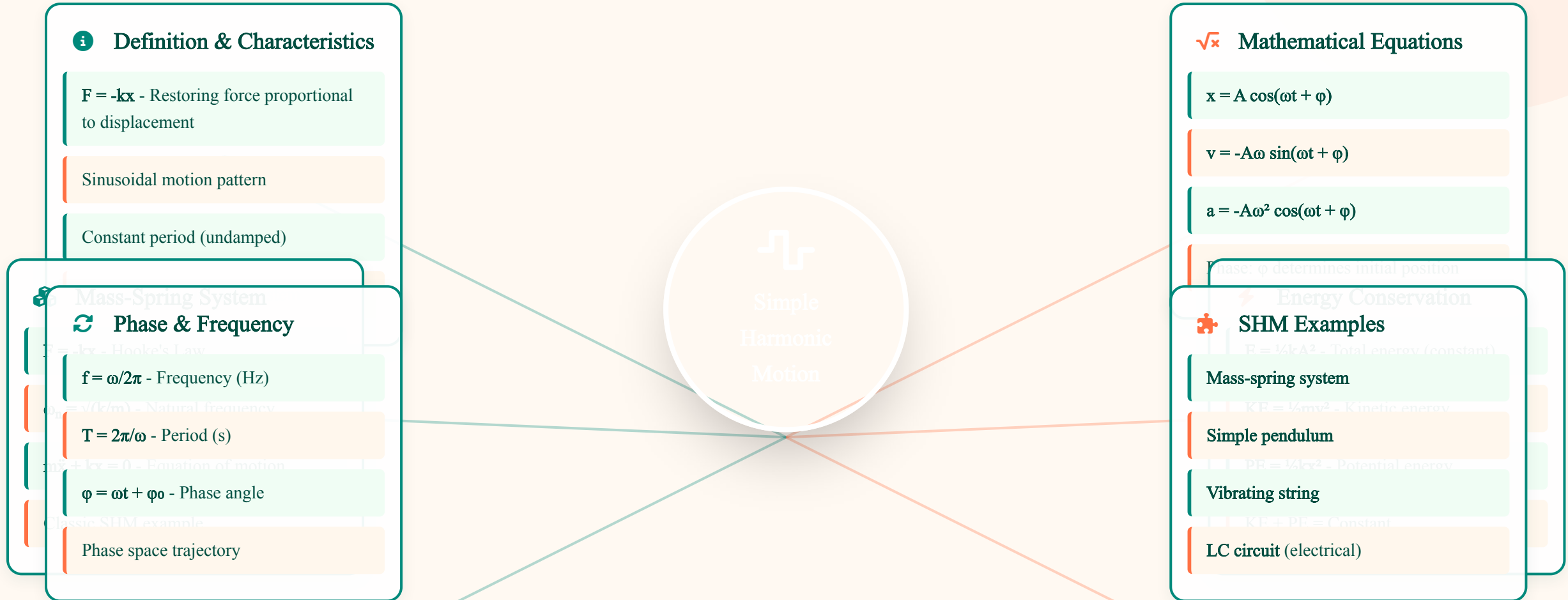
Constant Energy

Conservation

↔ Phase Space: Plots velocity vs. position

Simple Harmonic Motion Mind Map

GenAI-generated conceptual framework for SHM analysis



Recap & Summary

Essential SHM concepts for vibration analysis

Key SHM Concepts

Restoring force $F = -kx$

Sinusoidal motion

Constant period T

Energy conservation

Phase angle ϕ

Natural frequency ω_n

Key Equations

$$x = A \cos(\omega t + \phi)$$

$$v = -A\omega \sin(\omega t + \phi)$$

$$a = -A\omega^2 \cos(\omega t + \phi)$$

$$\omega_n = \sqrt{k/m}$$



Why SHM Matters

- ✓ Foundation for understanding all vibratory systems
- ✓ Essential for analyzing real-world engineering problems
- ✓ Basis for complex vibration analysis methods

Next Topics

Damped Harmonic Motion

Forced Vibration Analysis

Multi-degree-of-freedom Systems