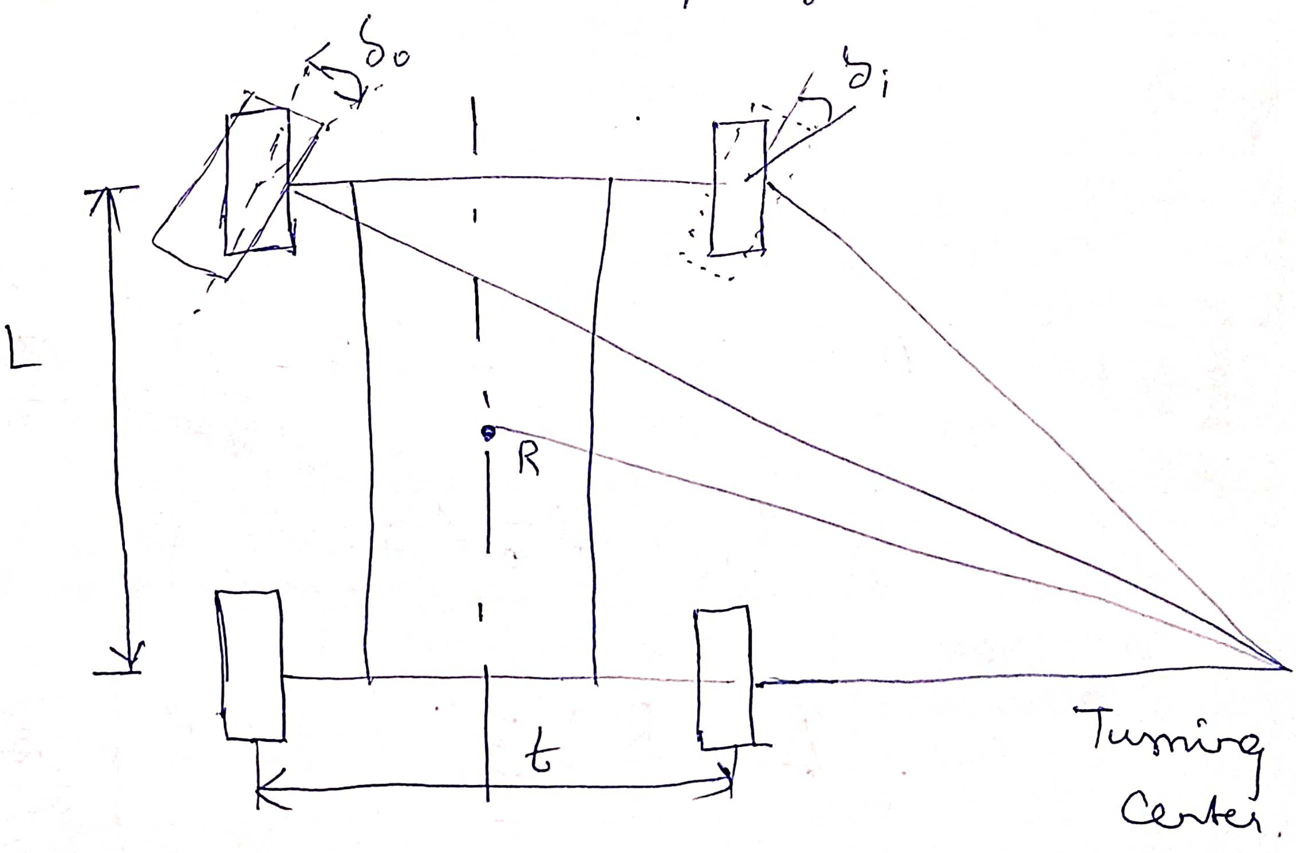
Steady State Cornering

Handling is the response of the vehicle to the driver input or the control of the driver. The driver and the vehicle is a closed loop System meaning that the driver know the vehicle direction or position of the vehicle to correct the given driver input to achieve the direction of Vehicle.

Open loop system meaning that the vehicle response to Specify Steering input. The most Commonly used System in understeer Gradient is Open loop System



$$\delta_o = \frac{L}{R+\frac{t}{2}} \qquad \delta_i = \frac{L}{R-\frac{t}{2}}.$$

$$\Delta = R\left(1-\cos\left(\frac{L}{R}\right)\right)$$

Consider $z = \frac{L}{R}$.

The above equation becomes

$$\Delta = R(1-\cos z) \rightarrow (1)$$

we know that

$$\cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} + \dots$$

$\rightarrow (2)$

Sub (2) in (1)

neglected.

$$\Delta = R\left(1-\left(1-\frac{z^2}{2!}+\frac{z^4}{4!}\right)\right)$$

$$\Delta = R\left[\cancel{1} \cancel{-1} + \frac{z^2}{2!}\right)$$

$$\Delta = R\left[\frac{z^2}{2}\right] \Rightarrow R\left[\frac{\left(\frac{L}{R}\right)^2}{2}\right]$$

$$\Delta = \cancel{R}\left[\frac{L^2}{R^{\cancel{2}} \times 2}\right]$$

$$\boxed{\Delta = \frac{L^2}{2R}}$$

Expression for Understeer gradient

We know that

$$F_y = C_\alpha \times \alpha \rightarrow (1)$$

where

F_y = Cornering force

C_α = Cornering Stifness.

α = Steer angle.

Cornering Coefficient

$$CC_\alpha = \frac{C_\alpha}{F_z} \rightarrow (2).$$

where F_z = load.

For a vehicle travelling ~~as~~ with a speed of B, the sum of forces in the lateral direction from the tyre must be equal to the mass time the centrifugal acceleration

$$\Sigma F_y = F_{y_f} + F_{y_r} = \frac{MV^2}{R} \rightarrow (3)$$

Where

F_{y_f} = Lateral force at front wheel

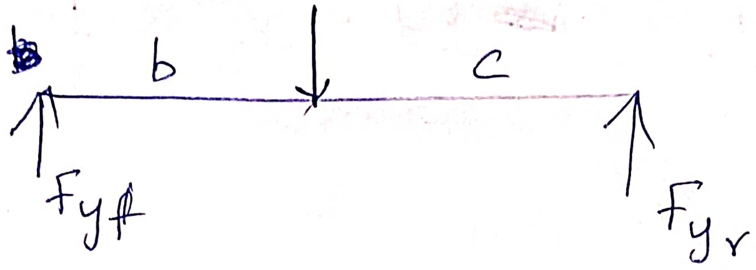
F_{y_r} = Lateral force at Rear wheel

M = Mass of the Vehicle

V = Speed of the Vehicle

R = Turning circle radius

The front wheel can be assumed by a steer angle with cornering force equal to both wheel, also for cornering vehicle, to be in a moment about center of gravity, the sum of the moment front and



$$F_{y_f} \times b = F_{y_r} \times c$$

$$F_{y_f} = F_{y_r} \times \frac{c}{b} \rightarrow (4)$$

Sub (4) in (3).

$$F_{y_f} + F_{y_r} = \frac{mv^2}{R}$$

$$F_{y_r} \times \frac{c}{b} + F_{y_r} = \frac{mv^2}{R}$$

$$F_{y_r}\left[\frac{c}{b} + 1\right] = \frac{mv^2}{R}$$

$$\boxed{b + c = L}$$

$$F_{y_r}\left[\frac{b+c}{b}\right] = \frac{mv^2}{R}$$

$$F_{y_r}\left[\frac{L}{b}\right] = \frac{mv^2}{R}$$

$$F_{y_r} = \frac{mv^2}{R} \times \frac{b}{L}$$

$$F_{y_r} = \frac{W}{g} \times \frac{V^2}{R} \times \frac{b}{L} \rightarrow (5)$$

Sub (5) in (4)

$$F_{y_f} = F_{y_r} \times \frac{c}{b}$$

$$= \frac{W}{g} \times \frac{V^2}{R} \times \frac{\cancel{b}}{L} \times \frac{c}{\cancel{b}}$$

$$F_{y_f} = \frac{W}{g} \times \frac{V^2}{R} \times \frac{c}{L} \rightarrow (6)$$

from (1)

$$F_y = C_\alpha \times \alpha$$

↳ write this equation in terms of front and rear

$$\alpha_f = \frac{F_{y_f}}{C'_{\alpha_f}}$$

$$= \frac{\frac{W}{g} \times \frac{V^2}{R} \times \frac{c}{L}}{C_{\alpha_f}}$$

replace $\frac{W \times c}{L} = W_f$

$$\alpha_r = \frac{F_{y_r}}{C_{\alpha_r}}$$

$$= \frac{\frac{W}{g} \times \frac{V^2}{R} \times \frac{b}{L}}{C_{\alpha_r}}$$

replace $\frac{W \times b}{L} = W_r$

$$\alpha_f = \frac{W_f \times V^2}{C_{\alpha f} \times R \times g} \rightarrow (7)$$

$$\alpha_r = \frac{W_r V^2}{C_{\alpha r} \times g R} \rightarrow (8)$$

$$\theta = \alpha_r + \delta_f - \alpha_f \rightarrow (9)$$

WKT $\theta = \frac{L}{R} \rightarrow (10)$

Sub (10), (7), (8) in (9)

$$\frac{L}{R} = \frac{W_r V^2}{C_{\alpha f} \times g \times R} + \delta_f - \frac{W_f V^2}{C_{\alpha r} \times R g}$$

$$\delta_f = \frac{L}{R} - \frac{W_r V^2}{C_{\alpha f}\, g R} + \frac{W_f V^2}{C_{\alpha r}\, g R}$$

$$= \frac{L}{R} + \left[\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \right] \frac{V^2}{Rg}$$

$$\boxed{\delta_f = \frac{L}{R} + K_{us} \cdot \frac{V^2}{Rg}}$$

where K_{us} is understeer gradient

$$K_{us} = \left[\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \right]$$