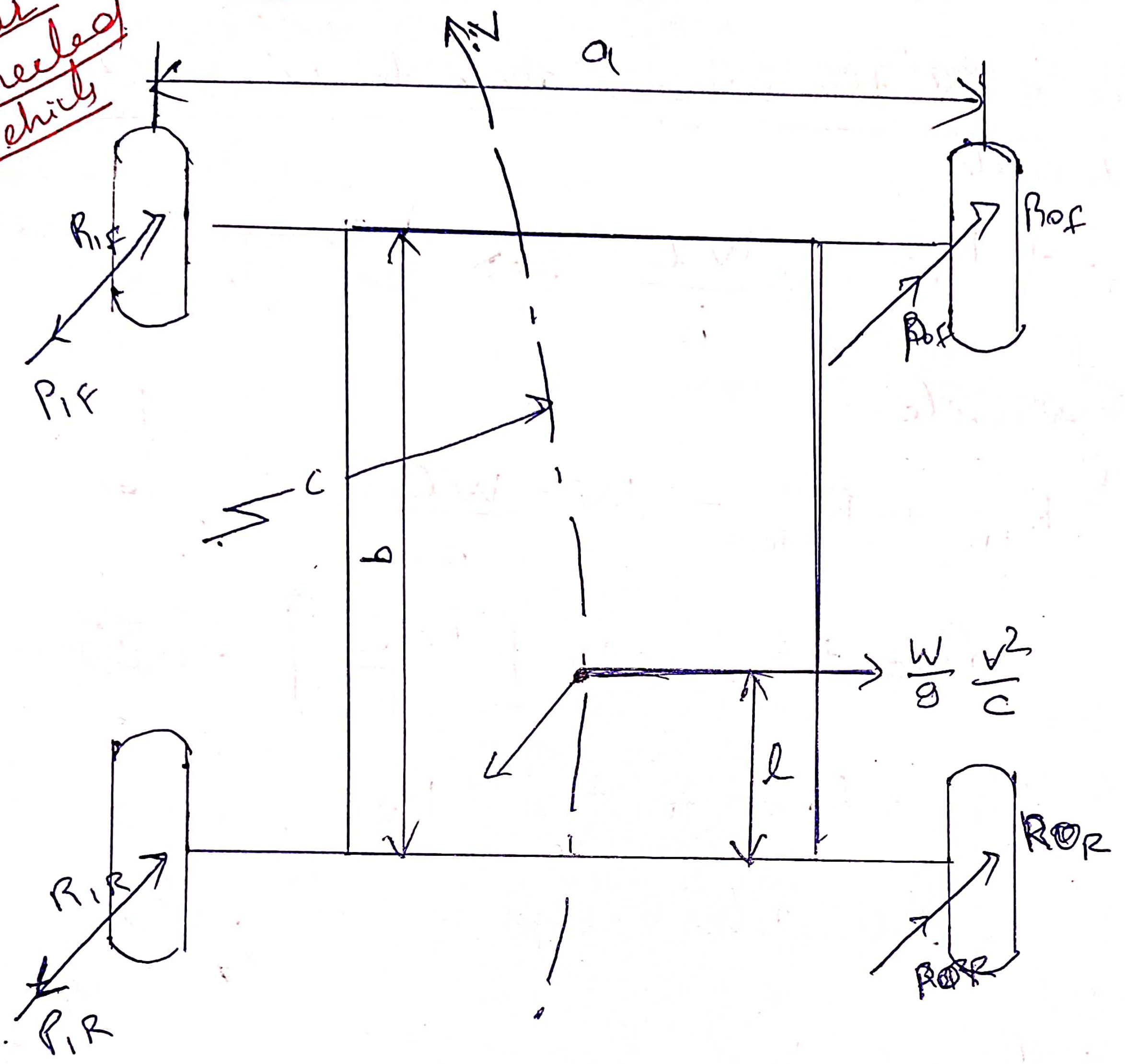
Stability of a Vehicle while taking turn

Four wheeled Vehicle



Let a vehicle take a turn to the left

Where, c = radius of curved path measured at CG of the vehicle

r = wheel radius

a = wheel track

b = wheel base.

h = height of CG of the Vehicle from ground

l = distance of CG in front of rear axle axis

V = linear speed of the Vehicle on road

W = Weight of the Vehicle

(i) Reaction at the wheel due to Weight

In front axle

$$R_{IF} + R_{OF} = \frac{Wl}{b} \rightarrow (1)$$

In Rear axle

$$R_{IR} + R_{OR} = W - \frac{Wl}{b}.$$

$$R_{IR} + R_{OR} = W\left[1 - \frac{l}{b}\right] \rightarrow (2)$$

Since $R_{IF} = R_{OF}$ & $R_{IR} = R_{OR}$

Sub above eqn in (1) & (2).

$$R_{IF} = R_{OF} = \frac{Wl}{2b} \rightarrow (3)$$

$$R_{IR} = R_{OR} = \frac{W}{2}\left[1 - \frac{l}{b}\right] \rightarrow (4)$$

(ii) Reaction at wheel due to Centrifugal force.

$$P_{IF} + P_{IR} = P_{OF} + P_{OR} = \frac{W}{g} \frac{V^2}{c} \frac{h}{a}.$$

Now, $P_{IF} = P_{OF} = \frac{Wl}{2b} \frac{v^2}{gc} \frac{h}{a}$

$P_{IR} = P_{OR} = \frac{W}{g} \left[1 - \frac{l}{b}\right] \frac{v^2}{gc} \frac{h}{a}.$

(iii) <u>Reaction at wheels due to Gyroscopic effect</u>

The magnitude of applied gyroscopic couple
$= I\omega \omega_p$

<u>Case (a):</u>

$\omega_s = \frac{V}{r}$ $\omega_f = \frac{GV}{r}$

$$\Sigma I\omega = I_s \frac{V}{r} + I_f \frac{GV}{r}$$
$$= \frac{V}{r}(I_s + GI_f)$$

Angular velocity of Precession, $\omega_p = \frac{V}{c}$

Gyroscopic precession $= \frac{V^2 (I_s + GI_f)}{crg}$

$$Q_{IF} = Q_{OF} = Q_{IR} = Q_{OR} = \frac{V^2}{2crga}(I_s + GI_f)$$

Case b

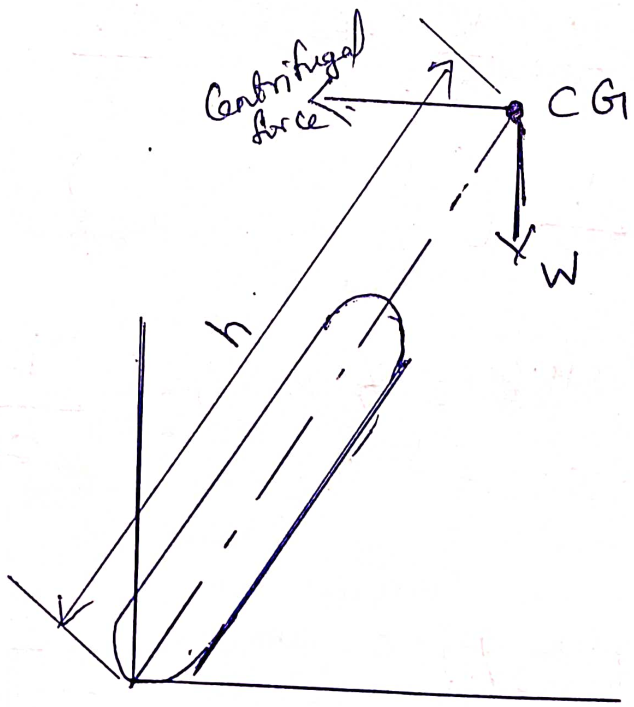
$$\Sigma I_w = I_f \frac{G V}{r} + I_s \frac{V}{r}$$

$$\text{Angular Velocity of Precession} = \frac{V}{c}$$

$$\text{Gyroscopic couple} = \frac{V^2}{crg}\left[I_f G + I_s\right]$$

$$\text{~~Effect~~ Reaction force} = \frac{V}{2crbg}\left[I_f G + I_s\right]$$

Two Wheeled Vehicle



$\theta \rightarrow$ Angle of heel to vertical

The axis of spin is also inclined to the horizontal at the angle θ due to heel over.
Now resolving this spin vector into $\Sigma I\omega \cos\theta$ and $\Sigma I\omega \sin\theta$ parallel and perpendicular to the road.

The gyroscopic effect is only due to the precession of the Vector $\Sigma I\omega \cos\theta$

$$= \frac{V}{r}(I_s + G I_f)\cos\theta$$

Angular Velocity of Precession $\omega_p = \frac{V}{c}$

$$\text{Gyroscopic Couple} = \frac{V^2}{crg}(I_s + G I_f)\cos\theta$$

Overturning couple due to Centrifugal force $= \frac{W}{g}\frac{V^2}{c} h \cos\theta.$

Therefore, the total overturning couple $= \frac{V^2}{cg}\left(\frac{I_s + G I_f}{r} + Wh\right)\cos\theta$

The balancing couple to the above is due to the moment of weight

$$= Wh \sin\theta.$$

Thus for equilibrium

$$Wh \sin\theta = \frac{V^2}{cg}\left[\frac{I_s + G I_f}{r} + Wh\right] \cos\theta$$

$$Wh \frac{\sin\theta}{\cos\theta} = \frac{V^2}{cg}\left[\frac{I_s + G I_f}{r} + Wh\right]$$

$$Wh \tan\theta = \frac{V^2}{cg}\left[\frac{I_s + G I_f}{r} + Wh\right]$$

$$\tan\theta = \frac{V^2}{cg} \times \frac{1}{Wh}\left[\frac{I_s + G I_f}{r} + Wh\right]$$

$$\tan\theta = \frac{V^2}{cg}\left[\frac{I_s + G I_f}{Whr} + \frac{Wh}{Wh}\right]$$

$$\tan\theta = \frac{V^2}{cg}\left[1 + \frac{I_s + G I_f}{Whr}\right]$$