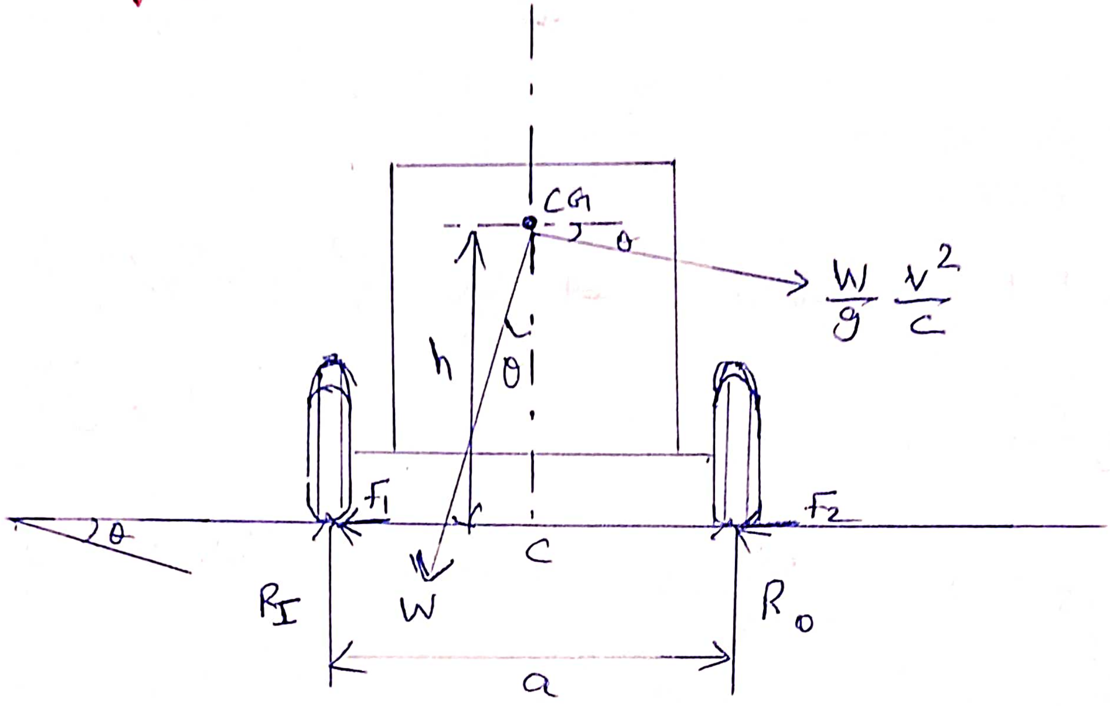
Stability of a Vehicle on a Banked Track



The forces acting are shown in this diagram

W = Weight of Vehicle

V = Velocity of Vehicle on Banked track

C = Radius of curved path measured at CG of vehicle

R_I and R_o = Normal reaction at inner and outer wheel respectively

F_I and F_o = Frictional forces at inner and outer wheels respectively

μ = Coefficient of adhesion between tyres and road surface

a = length of wheel track

θ = Inclination of wheel axes to horizontal

$\frac{W}{g}\frac{V^2}{c}$ = Centrifugal force acting at CG of Vehicle.

$\Sigma V = 0$ gives

$$R_o + R_I = W\cos\theta + \frac{W}{g}\frac{V^2}{c}\sin\theta \quad \text{(1)}$$

$\Sigma H = 0$ gives

$$F_I + F_o = \frac{W}{g}\frac{V^2}{c}\cos\theta - W\sin\theta \quad \text{(2)}$$

$\Sigma M_o = 0$ gives

$$(R_o - R_I)\frac{a}{2} = \frac{W}{g}\frac{V^2}{c}h\cos\theta - Wh\sin\theta$$

$$R_o - R_I = \frac{W}{g}\frac{V^2}{c}\frac{2h}{a}\cos\theta - \frac{W2h}{a}\sin\theta \quad \text{(3)}$$

from (1) & (3) we get

$$2R_o = W\cos\theta + \frac{W}{g}\frac{V^2}{c}\sin\theta + \frac{W}{g}\frac{V^2}{c}\frac{2h}{a}\cos\theta - \frac{W2h}{a}\sin\theta$$

$$2R_o = \frac{W}{g}\frac{V^2}{c}\left[\sin\theta + \frac{2h}{a}\cos\theta\right] + W\left[\cos\theta - \frac{2h}{a}\sin\theta\right]$$

$$R_0 = \frac{W}{2g}\frac{v^2}{c}\left[\sin\theta + \frac{2h\cos\theta}{a}\right] + \frac{W}{2}\left[\cos\theta - \frac{2h\sin\theta}{a}\right]$$

Sub R_0 in (1)

$$R_0 + R_I = W\cos\theta + \frac{W}{g}\frac{v^2}{c}\sin\theta.$$

$$\frac{W}{2g}\frac{v^2}{c}\left[\sin\theta + \frac{2h\cos\theta}{a}\right] + \frac{W}{2}\left[\cos\theta - \frac{2h\sin\theta}{a}\right]$$

$$+ R_I = W\cos\theta + \frac{W}{g}\frac{v^2}{c}\sin\theta.$$

$$R_I = W\cos\theta + \frac{W}{g}\frac{v^2}{c}\sin\theta. - \frac{W}{2g}\frac{v^2}{c}\left[\sin\theta + \frac{2h\cos\theta}{a}\right]$$

$$- \frac{W}{2}\left[\cos\theta - \frac{2h\sin\theta}{a}\right]$$

$$R_I = W\cos\theta + \frac{W}{g}\frac{v^2}{c}\sin\theta - \frac{Wv^2}{2gc}\sin\theta - \frac{W}{2g}\frac{v^2}{c}\frac{2h\cos\theta}{a}$$

$$- \frac{W}{2}\cos\theta + \frac{W}{2} \times \frac{2h\sin\theta}{a}$$

$$= \frac{W}{2}\cos\theta - \frac{W}{2g}\frac{v^2}{c}\frac{2h\cos\theta}{a} + \frac{W}{2g}\frac{v^2}{c}\sin\theta$$

$$+ \frac{W}{2} \times \frac{2h\sin\theta}{a}$$

$$R_I = \frac{W}{g}\frac{v^2}{c}\left[\frac{\sin\theta}{2} - \frac{h\cos\theta}{a}\right] + \frac{W}{2}\left[\cos\theta + \frac{2h\sin\theta}{a}\right]$$

If the vehicle begin to slide outward along the banking at the limited speed V_s then

$$F_I + F_O = \mu (R_I + R_O)$$

Sub eqn ① & ② in above equation.

$$F_I + F_O = \mu \left[W\cos\theta + \frac{W}{g}\frac{V_s^2}{c}\sin\theta \right]$$

$$\frac{W}{g}\frac{V_s^2}{c}\cos\theta - W\sin\theta = \mu\left[W\cos\theta + \frac{W}{g}\frac{V_s^2}{c}\sin\theta\right]$$

$$\frac{W}{g}\frac{V_s^2}{c}\cos\theta - W\sin\theta = \mu W\cos\theta + \frac{\mu W}{g}\frac{V_s^2}{c}\sin\theta$$

$$\frac{W}{g}\frac{V_s^2}{c}\cos\theta - \frac{\mu W}{g}\frac{V_s^2}{c}\sin\theta = \mu W\cos\theta + W\sin\theta$$

$$\frac{W}{g}\frac{V_s^2}{c}\left[\cos\theta - \mu\sin\theta\right] = W\left[\mu\cos\theta + \sin\theta\right]$$

$$\boxed{V_s^2 = \left[\frac{\mu\cos\theta + \sin\theta}{\cos\theta - \mu\sin\theta}\right] g\, c}$$

If V_o is the overturning speed of vehicle then putting $R_I = 0$ in eqn ① & ③ we get

$$W\cos\theta + \frac{W}{g}\frac{V_0^2}{c}\sin\theta = \frac{W}{g}\frac{V_0^2}{c}\frac{2h}{a}\cos\theta$$

$$- \frac{W 2h}{a}\sin\theta$$

$$\frac{W}{g}\frac{V_0^2}{c}\sin\theta - \frac{W}{g}\frac{V_0^2}{c}\frac{2h}{a}\cos\theta = -\frac{W2h}{a}\sin\theta - W\cos\theta$$

$$\frac{W}{g}\frac{V_0^2}{c}\left[\sin\theta - \frac{2h}{a}\cos\theta\right] = W\left[-\frac{2h}{a}\sin\theta - \cos\theta\right]$$

$$V_0^2 = \frac{\left[-\frac{2h}{a}\sin\theta - \cos\theta\right]}{\left[\sin\theta - \frac{2h}{a}\cos\theta\right]} \times cg$$

$$= \frac{\frac{-2h\sin\theta - a\cos\theta}{a}}{\frac{a\sin\theta - 2h\cos\theta}{a}} \times cg.$$

$$V_0^2 = \frac{-2h\sin\theta - a\cos\theta}{a\sin\theta - 2h\cos\theta} \times cg.$$

$$V_0^2 = \frac{-(2h\sin\theta + a\cos\theta)}{-(2h\cos\theta - a\sin\theta)} \times cg$$

$$\boxed{V_0^2 = \frac{2h\sin\theta + a\cos\theta}{2h\cos\theta - a\sin\theta} \times cg}$$