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DEPARTMENT OF AEROSPACE ENGINEERING

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Year & Branch	:	**III Aerospace**	Semester	:	**V**
Course	:	**19ASB302 – Finite Element Method for Aerospace**			

Unit:1
Solid mechanics stress equilibrium equation, strain-displacement equations

Stress equilibrium equation
Stresses is a continuous function of the location in a body. Therefore, the stress at any point is interrelated with the stress at other points. When force is acting along the x-axis of a solid, normal stress vis developed at the contacting surface.

The solid is a continuum and internal force is exerted by the contacting particles on the others. As a result, stress develops in the entire solid body rather than just the contacting point. Considering the rate of stress development per unit length along x-axis as the increment in stress across the length dx is:

$$\Delta\sigma_x = \frac{\partial\sigma_x}{\partial x}dx$$

If the point on solid is at rest and in equilibrium, the stress developed along the x- axis due to the aforementioned external force and internal force will be balanced by a stress of the same magnitude but in a different direction. Therefore, a'x can be expressed in term of a_x.

$$\sigma'_x = \sigma_x + \frac{\partial\sigma_x}{\partial x}dx$$

Similarly, the following normal stress components can be defined based on

$$\sigma'_y = \sigma_y + \frac{\partial\sigma_y}{\partial y}dy$$

$$\sigma'_z = \sigma_z + \frac{\partial\sigma_z}{\partial z}dz$$

Six shear stress components can be expressed in a similar fashion:

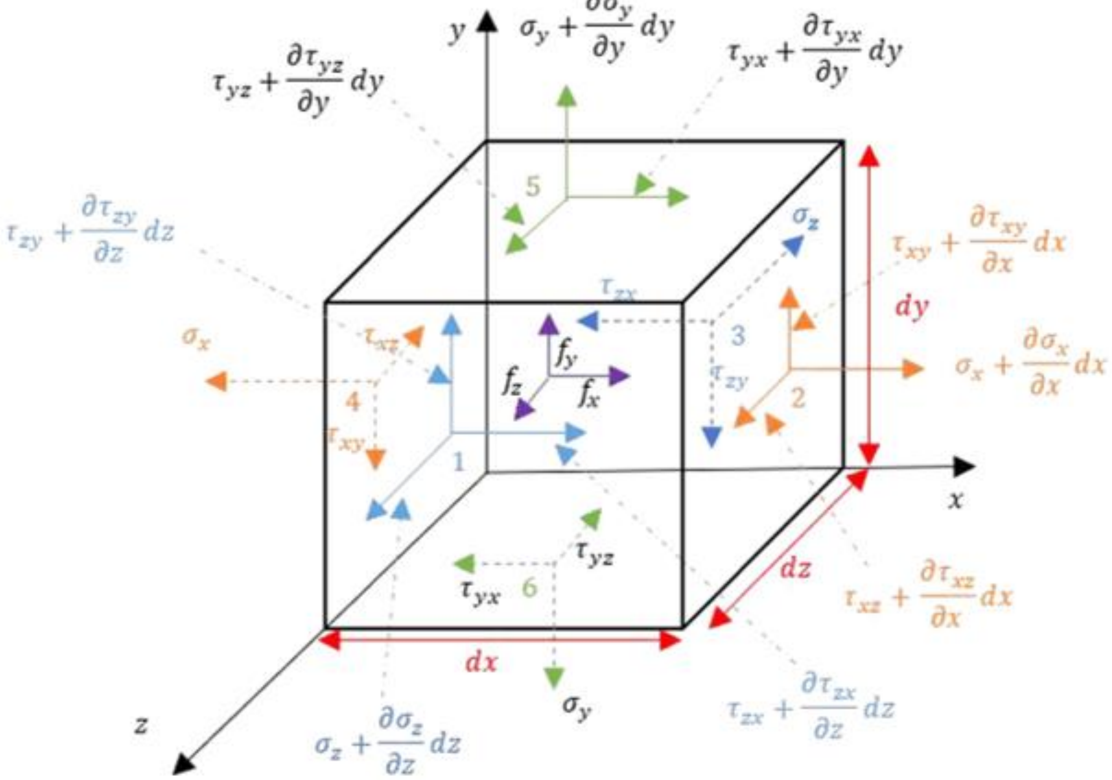
$$\tau'_{xy} = \tau_{xy} + \frac{\partial \tau_{xy}}{\partial x}dx \; \tau'_{yx} = \tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}dy$$

$$\tau'_{xz} = \tau_{xz} + \frac{\partial \tau_{xz}}{\partial x}dx \; \tau'_{zx} = \tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}dz$$

$$\tau'_{yz} = \tau_{yz} + \frac{\partial \tau_{yz}}{\partial y}dy \; \tau'_{zy} = \tau_{zy} + \frac{\partial \tau_{zy}}{\partial z}dz$$

A total of 18 stress components can be expressed in the above-derived forms. All these unknown stress components can now be determined by knowing only nine of them.

Other than the stresses developed on six planes, body forces may exist too. Let fx,fy and/; be the body forces (force per volume) acting along the x, y and z axes, as shown in Fig.

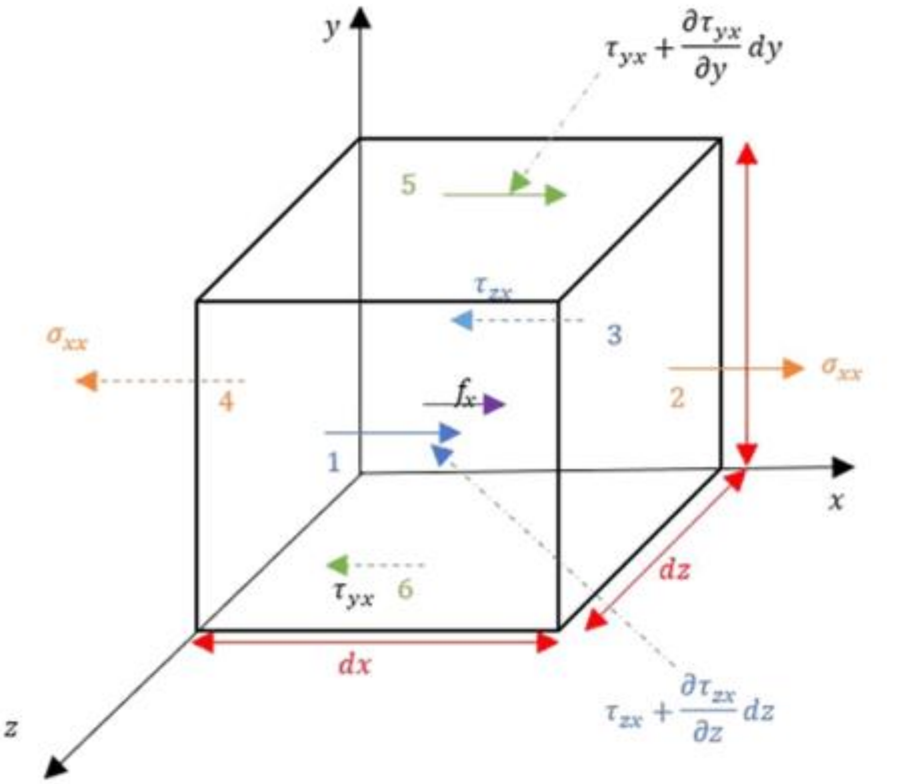


Components of stress in 3-D.

Under static equilibrium, the summation of forces acting on an infinitesimal part of the solid, as shown in Fig., along the x-axis is zero. The involved forces are shown in Fig.

$$\overset{+}{\rightarrow} \sum F_x = 0$$

$$\left[\sigma_x + \frac{\partial \sigma_x}{\partial x} dx\right] dydz - \sigma_x dydz + \left[\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} dy\right] dxdz - \tau_{yx} dxdz + \left[\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} dz\right] dxdy - \tau_{zx} dxdy + f_x dxdydz = 0$$



Stresses that act along .v-axis.

$$\sigma_x dydz + \frac{\partial \sigma_x}{\partial x} dxdydz - \sigma_x dydz + \tau_{yx} dxdz + \frac{\partial \tau_{yx}}{\partial y} dxdydz - \tau_{yx} dxdz + \tau_{zx} dxdy + \frac{\partial \tau_{zx}}{\partial z} dxdydz - \tau_{zx} dxdy + f_x dxdydz = 0$$

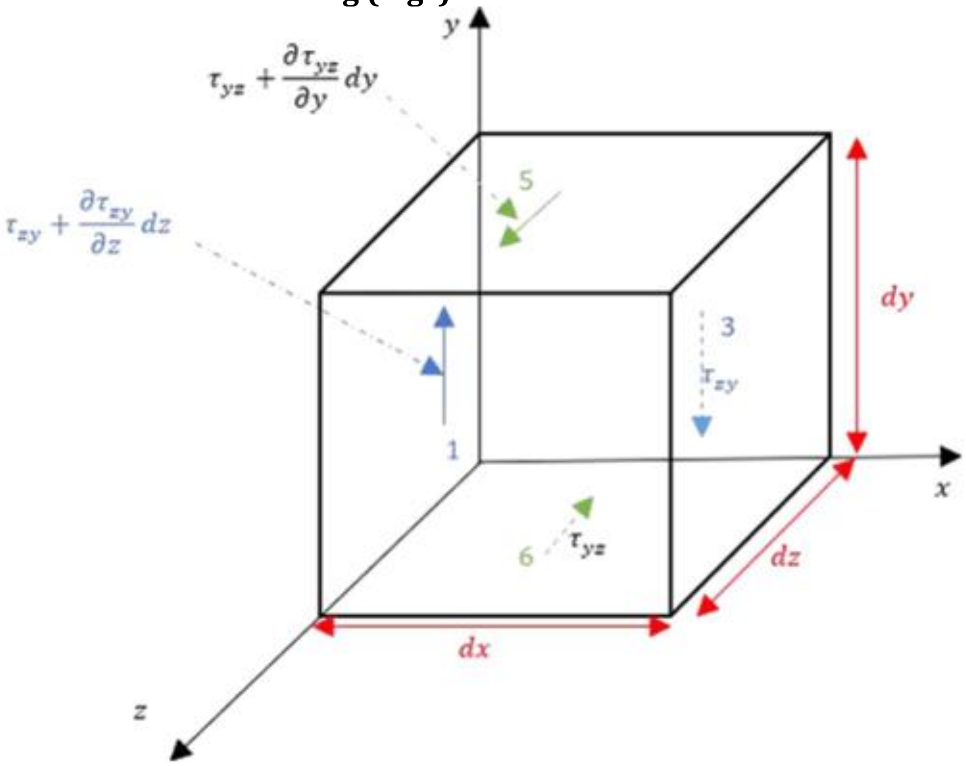
$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + f_x = 0$$

Similarly, equating the summation of forces along y and z axes to zero yields the following

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + f_y = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \tau_z}{\partial z} + f_z = 0$$

Eqs. above are equilibrium equation for stresses considering body forces. Equating the summation of moment about the л-axis induced by forces (Fig.) to zero results in the following (Fig.):



Stresses with the ability to cause rotation about x-axis.

$$\circlearrowleft + (\Sigma M_O)_x = 0$$

$$\left[\tau_{yz} + \overbrace{\frac{\partial \tau_{yz}}{\partial y} dy}^{\text{too small}}\right](dxdz)\left(\frac{dy}{2}\right) + \tau_{yz}(dxdz)\left(\frac{dy}{2}\right) - \left[\tau_{zy} + \underbrace{\frac{\partial \tau_{zy}}{\partial z} dz}_{\text{too small}}\right](dxdy)\left(\frac{dz}{2}\right)$$
$$- \tau_{zy}(dxdy)\left(\frac{dz}{2}\right) = 0$$

Neglecting the 4th order products, because they are insignificant, gives the following:

$$\tau_{yz} dxdydz - \tau_{zy} dxdydz = 0$$

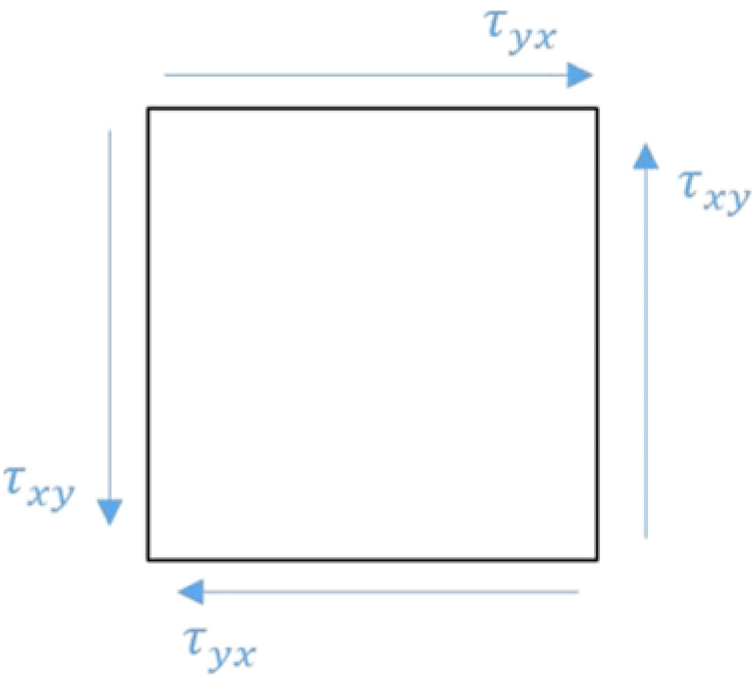
$$\tau_{yz} = \tau_{zy}$$

Similarly, the following relationships are produced by taking the summation of moment about the v and z axes as zero:

$$\tau_{xy} = \tau_{yx}$$

$$\tau_{zx} = \tau_{xz}$$

Eqs. above show that for each pair of shear stress (say r*v) developed. a complementary shear stress (say rvv) of the same magnitude will also be developed on their adjacent face. These complementary shear stresses act in the direction that can stabilise the rotation that would be caused by the developed shear stress pair alone, as shown in Fig.



Complementary pairs of shear stresses.

Therefore, the stress components in Eq. can be expressed as a symmetrical matrix:

$$\sigma = \begin{bmatrix} \sigma_X & \tau_{XY} & \tau_{XZ} \\ \tau_{XY} & \sigma_Y & \tau_{YZ} \\ \tau_{ZX} & \tau_{YZ} & \sigma_Z \end{bmatrix}$$

Now, the stress at a point is defined completely by six independent components (three normal and three shear) instead of nine independent components (three normal and six shear).

Prepared: Dr. M. Subramanian/Professor & Head Aerospace Engineering