# SNS COLLEGE OF TECHNOLOGY

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## DEPARTMENT OF AEROSPACE ENGINEERING

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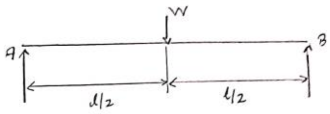
| Faculty Name | : | **Dr.M.Subramanian, Prof & Head/ Aerospace** | Academic Year | : | **2024-2025 (Odd)** |
| Year & Branch | : | **III Aerospace** | Semester | : | **V** |
| Course | : | **19ASB302 – Finite Element Method for Aerospace** | | | |

Unit: 1

A simply supported beam carries a point load (w) at midspan of beam. Determine the deflection using Rayleigh ritz method and Calculate the deflection at midspan. Also find the bending moment.

The fourier series equation for simply supported beam




$$y = \sum_{n=1,2,5}^{\infty} a \sin\frac{n\pi x}{l} \quad ; \quad a \to \text{rayleigh ritz parameter}$$

$$y = a_1 \sin\frac{\pi x}{l} + a_2 \sin\frac{3\pi x}{l} \quad \text{—(1)}$$

The total potential energy, $\pi = U - H$

The strain energy of beam due to bending is

$$U = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx.$$

Diff eq (1) w.r.t x

$$\Rightarrow \frac{dy}{dx} = a_1 \cos\frac{\pi x}{l}\cdot\frac{\pi}{l} + a_2 \cos\frac{3\pi x}{l}\,\frac{3\pi}{l}$$

$$= a_1\frac{\pi}{l}\cos\frac{\pi x}{l} + \frac{a_2 3\pi}{l}\cos\frac{3\pi x}{l}$$

$$\frac{d^2y}{dx^2} = -\frac{a_1\pi}{l}\sin\frac{\pi x}{l}\left(\frac{\pi}{l}\right) - \frac{a_2 3\pi}{l}\sin\frac{3\pi x}{l}\left(\frac{3\pi}{l}\right)$$

$$= \frac{-a_1\pi^2}{l^2}\sin\frac{\pi x}{l} - \frac{a_2 9\pi^2}{l}\sin\frac{3\pi x}{l} \quad \text{—(3)}$$

Sub (3) in (1)

$$\therefore U = \frac{EI}{2}\int_0^l\left[\frac{-a_1\pi^2}{l^2}\sin\frac{\pi x}{l} - \frac{a_2 9\pi^2}{l}\sin\frac{3\pi x}{l}\right]^2 dx$$

$$U = \frac{EI\pi^4}{2l^4}\int_0^l\left[a_1\sin\frac{\pi x}{l} + 9a_2\sin\frac{3\pi x}{l}\right]^2 dx$$

$$U = \frac{EI\pi^4}{2l^4}\int_0^l\left[a_1^2\sin^2\frac{\pi x}{l} + 81a_2^2\sin^2\frac{3\pi x}{l} + 18a_1\sin\frac{\pi x}{l}\,a_2\sin\frac{3\pi x}{l}\right] dx$$

$$U = \frac{EI\pi^4}{2l^4}\int_0^l \left[ a_1^2 \left( \frac{1-\cos\frac{2\pi x}{l}}{2} \right) dx + 81a_2^2 \left( \frac{1-\cos\frac{6\pi x}{l}}{2} \right) dx + \frac{18a_1a_2\left[\cos\left(\frac{\pi x}{l}-\frac{3\pi x}{l}\right) - \cos\left(\frac{\pi x}{l}+\frac{3\pi x}{l}\right)\right] dx}{2} \right]$$

$$U = \frac{EI\pi^4}{2l^4}\left[ \frac{a_1^2}{2}\left( x - \frac{\sin\frac{2\pi x}{l}}{\frac{2\pi}{l}} \right)_0^l + \frac{81a_2^2}{2}\left( x - \frac{\sin\frac{6\pi x}{l}}{6\pi/l} \right)_0^l + 9a_1a_2\left[ \frac{\sin\frac{2\pi x}{l}}{2\pi/l} - \frac{\sin\frac{4\pi x}{l}}{\frac{4\pi}{l}} \right]_0^l \right.$$

$$U = \frac{EI\pi^4}{2l^4}\left[ \frac{a_1^2}{2}\left[ l - \frac{\sin\frac{2\pi l}{l}}{2\pi/l} - 0 + \frac{\sin 0}{\frac{2\pi}{l}} \right] + \frac{81a_2^2}{2}\left[ l - \frac{\sin\frac{6\pi l}{l}}{\frac{6\pi}{l}} - 0 + \frac{\sin 0}{\frac{6\pi}{l}} \right] \right.$$

$$+ 9a_1a_2\left[ \frac{\sin\frac{2\pi l}{l}}{2\pi/l} - \frac{\sin\frac{4\pi l}{l}}{4\pi/l} - \frac{\sin\frac{2\pi(0)}{l}}{2\pi/l} \right] + \left. \frac{\sin\frac{4\pi(0)}{l}}{4\pi/l} \right]$$

Workdone by external force

$$H = W y_{max}$$

$$y = a_1 \sin\frac{\pi x}{l} + a_2 \sin\frac{3\pi x}{l}$$

At $x = \frac{l}{2}$; $y_{max} = a_1 \sin\frac{\pi(l/2)}{l} + a_2 \sin\frac{3\pi(l/2)}{l}$

$$= a_1 \sin\frac{\pi l}{2l} + a_2 \sin\frac{3\pi l}{2l}$$

$$y_{max} = a_1 - a_2 \qquad \therefore H = W(a_1 - a_2)$$

Total potential energy: $\pi = U - H$

$$\Rightarrow \pi = \frac{EI\pi^4}{4l^3}\left(a_1^2 + 81a_2^2\right) - W(a_1 - a_2)$$

For stationary value of $\pi$, the following conditions must be satisfied

$$\frac{\partial \pi}{\partial a_1} = 0 \; ; \; \frac{\partial \pi}{\partial a_2} = 0$$

Diff $\pi$ w.r.t $a_1$

$$\therefore \frac{\partial \pi}{\partial a_1} = \frac{EI\pi^4}{4l^3}(2a_1 + 0) - W(1) = 0$$

2/3

$$\Rightarrow a_1 = \frac{2l^3 W}{EI\pi^4}$$

Diff $\pi$ w.r.t $a_2$

$$\therefore \frac{\partial \pi}{\partial a_2} = \frac{EI\pi^4}{4l^3}(162a_2) - W(-1) = 0$$

$$\Rightarrow \frac{EI\pi^4}{4l^3}(162a_2) = -W$$

$$\Rightarrow a_2 = \frac{-2l^3 W}{81EI\pi^4}$$

Sub $a_1$ and $a_2$ in ① $y_{max} = \frac{2l^3 W}{EI\pi^4} + \frac{2l^3 W}{81EI\pi^4}$

$$\Rightarrow y_{max} = \frac{2.03Wl^3}{EI\pi^4}$$

The exact solution for a simply supported Beam carrying a point load at midpoint will be,

$$y = \frac{Wl^3}{48EI}$$

Bending moment $M = EI\frac{d^2y}{dx^2}$

$$\Rightarrow M = EI\left[\frac{-a_1\pi^2}{l^2}\sin\frac{\pi x}{l} - a_2\frac{9\pi^2}{l^2}\sin\frac{3\pi x}{l}\right]$$

$$a_1 \,\&\, a_2 = -EI\left[\frac{2Wl^3\pi^2}{EI\pi^4 l^2}\sin\frac{\pi x}{l} - 18\frac{Wl^3\pi^2}{81EI\pi^4 l^2}\sin\frac{3\pi x}{l}\right]$$

put $x = l/2$

$$M = -EI\left[\frac{2Wl}{EI\pi^2}\sin\frac{\pi}{2} - \frac{2Wl}{9EI\pi^2}\sin\frac{3\pi}{2}\right]$$

$$M = -EI\left[\frac{2Wl}{EI\pi^2}(1) - \frac{2Wl}{9EI\pi^2}(-1)\right]$$

$$M = -EI\left[1.78\frac{Wl}{EI\pi^2}\right]$$

$$M = -0.225Wl$$