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COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF AEROSPACE ENGINEERING

Faculty Name	:	**Dr.M.Subramanian, Prof & Head/ Aerospace**	Academic Year	:	**2024-2025 (Odd)**
Year & Branch	:	**III Aerospace**	Semester	:	**V**
Course	:	**19ASB302 – Finite Element Method for Aerospace**			

Unit: 1

The differential equation of a physical phenomenon is given by $\frac{d^2y}{dx^2}+500x^2=0,\ 0\le x\le 1$, Trial function $y=a_1(x-x^3)+a_2(x-x^5)$, Calculate the value of parameters, a_1 and a_2. By the following methods. (i) Point collocation method (ii) Sub-domain collocation method (iii) Least Square Method and (iv) Galerkin's method. Boundary condition are, y (0)=0, y(1)=0

Given: Differential equation

$$\frac{d^2y}{dx^2}+500x^2=0,\quad 0\le x\le 1 \qquad \rightarrow (1)$$

Trial function, $y=a_1(x-x^2)+a_2(x-x^5)$

Boundary condition are $y(0)=0,\ y(1)=0$

To find: The value of the parameter a_1 and a_2 by

(i) point collocation method

(ii) Subdomain collocation method

(iii) Least Squares method

(iv) Galerkin's method

Sol: First we have to verify, whether the trial function satisfies the boundary conditions or not.

Trial function is $y=a_1(x-x^3)+a_2(x-x^5)$

When $x=0,\ y=0$ $\quad x=1,\ y=0$

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Hence it satisfies the boundary conditions.

Residual, R: $y = a_1(x - x^5) + a_2(x - x^5)$

$$\frac{dy}{dx} = a_1(1 - 3x^2) + a_2(1 - 5x^4)$$

$$\frac{d^2y}{dx^2} = a_1(-6x) + a_2(-20x^3)$$

$$\frac{d^2y}{dx^2} = -6a_1x - 20a_2x^3$$

Substitute $\frac{d^2y}{dx^2}$ value in given differential equation (1)

$\Rightarrow$ Residual, $R = -6a_1x - 20a_2x^3 + 500x^2$ → (2)

The interval 0 to 1 is divided into two domains 0 to $\frac{1}{2}$ and $\frac{1}{2}$ to 1.

(i) point collocation: In point collocation method residual are set to zero.

$\Rightarrow R = -6a_1x - 20a_2x^3 + 500x^2 = 0$ → (3)

Domain (i): Limit is 0 to $\frac{1}{2}$, In domain (1), we can choose an arbitrary point. Let it be 1/3

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So, put $x = \frac{1}{3}$ in equation (3)

$$R = -6a_1\left[\frac{1}{3}\right] - 20a_2\left[\frac{1}{3}\right]^3 + 500\left[\frac{1}{3}\right]^2 = 0$$

$$-2a_1 - \frac{20}{27}a_2 + \frac{500}{9} = 0$$

$$-2a_1 - 0.741a_2 = -55.55$$

$$2a_1 + 0.741a_2 = 55.55$$

$$a_1 + 0.3705a_2 = 27.775 \quad \text{—④}$$

Domain (2): Limit in $\frac{1}{2}$ to 1: In domain (2 we can choose $x = 2/3$ and substituting the same in equation (3).

$$R = -6a_1\left[\frac{2}{3}\right] - 20a_2\left[\frac{2}{3}\right]^3 + 500\left[\frac{2}{3}\right]^2 = 0$$

$$-4a_1 - 20a_2 \times \frac{8}{27} + 50 \times \frac{4}{9} = 0$$

$$-4a_1 - 5.925a_2 + 222.22 = 0$$

$$4a_1 + 5.925a_2 = 222.22$$

$$a_1 + 1.481a_2 = 55.555 \quad \text{—⑤}$$

Solving equation (4) & (5),

$$-a_1 - 0.3705a_2 = -27.775$$

$$a_1 + 1.481a_2 = 55.555$$

$$1.111a_2 = 27.78$$

$$a_2 = 27.78/1.111 \Rightarrow a_2 = 25$$

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Subsitute a_2 Value in equation (4) or (5).

(5) $\Rightarrow$ $a_1 + 1.481(25) = 55.555$

$a_1 + 37.025 = 55.555$

$a_1 = 18.53$

Hence the trial function is, $y = 18.53(x - x^3) + 25(x - x^5)$

[ii] Sub domain Collocation:

This method requires, $\int_0^1 R\,dx = 0$

The interval 0 to 1 is divided into two domain 0 to $\frac{1}{2}$ and $\frac{1}{2}$ to 1

For domain (1) $\int_0^{1/2} R\,dx = 0$

Substitute R Value,

$$\int_0^{\frac{1}{2}} [-6a_1 x - 20a_2 x^3 + 500x^2]\,dx = 0$$

$$-6a_1\left[\frac{x^2}{2}\right]_0^{1/2} - 20a_2\left[\frac{x^4}{4}\right]_0^{1/2} + 500\left[\frac{x^3}{3}\right]_0^{1/2} = 0$$

$$-6a_1\left[(1/2)^2 - 0\right] - \frac{20a_2}{4}\left[(1/2)^4 - 0\right] + \frac{500}{3}\left[(1/2)^3 - 0\right] = 0$$

$$-\frac{6a_1}{8} - \frac{20a_2}{64} + \frac{500}{24} = 0$$

$$-0.75a_1 - 0.3125a_2 + 20.83 = 0$$

$$0.75a_1 + 0.3125a_2 = 20.82$$

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$a_1 + 0.4166a_2 = 27.773$ ---(6)

For domain (2) $\int_1^{9/2} R\,dx = 0$

Substitute R Value

$\int_1^{1/2} [-6a_1 x - 20a_2 x^3 + 500x^2]\,dx = 0$

$-6a_1\left[\frac{x^2}{2}\right]_{1/2}^{1} - 20a_2\left[\frac{x^4}{4}\right]_{1/2}^{1} + 500\left[\frac{x^3}{3}\right]_{1/2}^{1} = 0$

$\frac{-6a_1}{2}\left[1-\left(\frac{1}{2}\right)^2\right] - \frac{20a_2}{4}\left[1-\left(\frac{1}{2}\right)^4\right] + \frac{500}{3}\left[1-\left(\frac{1}{2}\right)^3\right] = 0$

$-\frac{6a_1}{2}[0.75] - \frac{20a_2}{4}[0.9375] + \frac{500}{3}[0.875] = 0$

$-2.25a_1 - 4.6875a_2 + 145.83 = 0$

$2.25a_1 + 4.6875a_2 = 145.83$

$a_1 + 2.083a_2 = 64.813$ ---(7)

Solving equation (6) & (7)

$-a_1 - 0.4166a_2 = -27.773$

$a_1 + 2.083a_2 = 64.813$

$1.6664a_2 = 37.04$

$a_2 = 22.23$

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Substitute a_2 value in equation (6) (or) 7

⑦ $\Rightarrow$ $a_1 + 2.083(22.23) = 64.813$

$a_1 + 46.305 = 64.813$

$a_1 = 18.50$

Hence the trial function is,

$y = 18.50(x - x^3) + 22.23(x - x^5)$

(iii) Least Squares method

This method requires $I = \int_0^1 R^2 dx$

It can be written as $\frac{\partial I}{\partial a_1} = \int_0^1 R \frac{\partial R}{\partial a_1} dx$ --- ⑧

For domain (1) $\frac{\partial I}{\partial a_1} = \int_0^{1/2} R \frac{\partial R}{\partial a_1} dx$ — ⑨

we know that, $R = -6a_1x - 20a_2x^3 + 500x^2$

$\frac{\partial R}{\partial a_1} = -6x$ $\vdash$ Substitute R, and $\frac{\partial R}{\partial a_1}$ values in equation ⑨

$\Rightarrow \frac{\partial I}{\partial a_1} = \int_0^{1/2} (-6a_1x - 20a_2x^3 + 500x^2)(-6x) dx$

The requirement is $\frac{\partial I}{\partial a_1} = 0$

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Unit: 1

$$\int_0^{1/2} [-6a_1x - 20a_2x^3 + 500x^2)(-6x)\,dx = 0$$

$$\int_0^{1/2} [-36a_1x^2 + 120a_2x^4 - 3000x^3]\,dx = 0$$

$$\frac{36a_1}{3}\left[\left(\frac{x^3}{3}\right)\right]_0^{1/2} + 120a_2\left[\frac{x^5}{5}\right]_0^{1/2} - 3000\left[\frac{x^4}{4}\right]_0^{1/2} = 0$$

$$\frac{36a_1}{3}\left[\left(\frac{1}{2}\right)^3 - 0\right] + \frac{120a_2}{5}\left[\left(\frac{1}{2}\right)^5 - 0\right] - 3000\left[\left(\frac{1}{2}\right)^4 - 0\right] = 0$$

$$12a_1\left[\frac{1}{8}\right] + 24a_2\left[\frac{1}{32}\right] - 750\left[\frac{1}{16}\right] = 0$$

$$1.5a_1 + 0.75a_2 = 46.875$$

$$a_1 + 0.5a_2 = 31.25 \quad \text{——— (10)}$$

For domain (2) $\dfrac{\partial I}{\partial a_2} = \int_{1/2}^{1} R\,\dfrac{\partial R}{\partial a_2}\,dx$ ---(11)

We know that, $R = -6a_1x - 20a_2x^3 + 500x^2$

$\dfrac{\partial R}{\partial a_2} = -20x^3$

Substitute R and $\dfrac{\partial R}{\partial a_2}$ Values in equation (11)

$$\frac{\partial I}{\partial a_2} = 0 \Rightarrow \int_{1/2}^{1} [-6a_1x - 20a_2x^3 + 500x^2][-20x^3] = 0$$

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Unit: 1

$$\int_{\frac{1}{2}}^{1} [-6a_1x - 20a_2x^3 + 500x^2][-20x^3] = 0$$

$$\int_{\frac{1}{2}}^{1} [120a_1x^4 + 400a_2x^6 - 1000x^5] = 0$$

$$120a_1\left[\frac{x^5}{5}\right]_{1/2}^{1} + 400a_2\left[\frac{x^7}{7}\right]_{1/2}^{1} - 1000\left[\frac{x^6}{6}\right]_{1/2}^{1} = 0$$

$$\frac{120a_1}{5}\left[(1)^5 - \left(\frac{1}{2}\right)^5\right] + \frac{400a_2}{7}\left[(1)^7 - \left(\frac{1}{2}\right)^7\right] - \frac{1000}{6}\left[(1)^6 - \left(\frac{1}{2}\right)^6\right] = 0$$

$$24a_1[1 - 0.03125] + 57.142a_2[1 - 0.00781] - 1666.66[1 - 0.01562] = 0$$

$$23.25a_1 + 56.695a_2 - 1640.626 = 0$$

$$23.25a_1 + 56.695a_2 = 1640.626$$

$$a_1 + 2.4389a_2 = 70.564 \quad \text{---} (12)$$

Solving equation (10) and (12)

$$-a_1 - 0.5a_2 = -31.25$$

$$a_1 + 2.439a_2 = 70.564$$

$$1.938a_2 = 39.314$$

$$a_2 = 20.28$$

Substitute a_2 Value in equation (10) or (12)

(10) $\rightarrow$ $a_1 + 0.5(20.28) = 31.25$ $\rightarrow$ $a_1 = 21.11$

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Unit: 1

Hence the trial function is

$y = 21.11(x - x^3) + 20.28(x - x^5)$

[iv] Galerkin's Method:

In this method, the trial function itself is considered as the weighting function w_i

$$\int_0^1 w_i R\,dx = 0$$

For domain (1): $\int_0^{\frac{1}{2}} w_i R\,dx = 0$ — (13)

Here, the trial function is, $y = w_1 = x - x^3$

Residual, R value is, $R = -6a_1x - 20a_2x^3 + 500x^2$

Substitute w_i and R Values in equation (13)

$$\int_0^{\frac{1}{2}} (x - x^3)(-6a_1x - 20a_2x^3 + 500x^2)\,dx = 0$$

$$\int_0^{\frac{1}{2}} \left[-6a_1x^2 - 20a_2x^4 + 500x^3 + 6a_1x^4 + 20a_2x^6 - 500x^5\right]dx = 0$$

$$-6a_1\left[\frac{x^3}{3}\right]_0^{\frac{1}{2}} - 20a_2\left[\frac{x^5}{5}\right]_0^{\frac{1}{2}} + 500\left[\frac{x^4}{4}\right]_0^{\frac{1}{2}} + 6a_1\left[\frac{x^5}{5}\right]_0^{\frac{1}{2}} +$$

$$20a_2\left[\frac{x^7}{7}\right]_0^{\frac{1}{2}} - 500\left[\frac{x^6}{6}\right]_0^{\frac{1}{2}} = 0$$

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$$-2a_1\left[\left(\tfrac{1}{2}\right)^3-0\right]-4a_2\left[\left(\tfrac{1}{2}\right)^5-0\right]+125\left[\left(\tfrac{1}{2}\right)^4-0\right]+$$

$$10.2a_1\left[\left(\tfrac{1}{2}\right)^5-0\right]+20.857a_2\left[\left(\tfrac{1}{2}\right)^7-0\right]-83.33\left[\left(\tfrac{1}{2}\right)^6-0\right]=0$$

$$-2a_1(0.125)-4a_2(0.03125)+125(0.0625)+12a_1(0.03125)$$
$$+2.857a_2(0.00781)-83.33(0.0156)=0$$

$$-0.25a_1-0.125a_2+7.8125+0.03759a_1+0.02239a_2-1.299=0$$

$$-0.21259a_1-0.10279a_2+6.5135=0$$

$$0.2125a_1+0.10279a_2=6.5135$$

$$a_1+0.4832a_2=30.651 \quad \text{---(14)}$$

For domain (2): $\int_{1/2}^{1} W_2 R\,dx=0$ ---(15)

Here, the trial function is $y=w_2=(x-x^5)$

$R=-6a_1x-20a_2x^3+500x^2$

Substitute w_2 and R value in eq (15)

$$\int_{1/2}^{1}(x-x^5)\left[-6a_1x-20a_2x^3+500x^2\right]dx=0$$

$$\int_{1/2}^{1}-6a_1x^2-20a_2x^4+500x^3+6a_1x^6+20a_2x^8-500x=0$$

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$$-6a_1\left[\frac{x^3}{3}\right]_{\frac{1}{2}}^{1} - 20a_2\left[\frac{x^5}{5}\right]_{\frac{1}{2}}^{1} + 500\left[\frac{x^4}{4}\right]_{1/2}^{1} + 6a_1\left[\frac{x^7}{7}\right]_{1/2}^{1} +$$

$$20a_2\left[\frac{x^9}{9}\right]_{1/2}^{1} - 500\left[\frac{x^8}{8}\right]_{1/2}^{1} = 0$$

$$-2a_1\left[1-\left(\frac{1}{2}\right)^3\right] - 4a_2\left[1-\left(\frac{1}{2}\right)^5\right] + 125\left[1-\left(\frac{1}{2}\right)^4\right] +$$

$$0.8571a_1\left[1-\left(\frac{1}{2}\right)^7\right] + 2.224a_2\left[1-\left(\frac{1}{2}\right)^9\right] - 62.5\left[1-\left(\frac{1}{2}\right)^8\right] = 0$$

$$-1.75a_1 - 3.875a_2 + 117.187 + 0.850a_1 + 2.215a_2 - 62.255 = 0$$

$$-0.9a_1 - 1.659a_2 + 54.932 = 0$$

$$0.9a_1 + 1.659a_2 = 54.932$$

$$a_1 + 1.843a_2 = 61.033 \quad \text{—— (16)}$$

Solving equation (14) and (16)

$$-a_1 - 0.4832a_2 = -30.651$$

$$a_1 + 1.843a_2 = 61.035$$

$$1.3598a_2 = 30.384$$

$$a_2 = 22.34$$

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Substitute a_2 value in equation (16)

$a_1 + 1.843(22.34) = 61.035$

$a_1 + 41.173 = 61.035$

$a_1 = 19.862$

Hence the trial function is

$y = 19.862(x - x^3) + 22.34(x - x^5)$

Result:

	a_1	a_2
(i) point Collocation	18.53	25
(ii) Subdomain Collocation	18.50	22.23
(iii) Least Squares method	21.11	20.28
(iv) Galerkin's method	19.862	22.34