SNS COLLEGE OF TECHNOLOGY

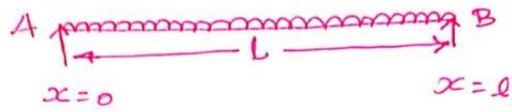
(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)
COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF AEROSPACE ENGINEERING

Faculty Name	:	**Dr.M.Subramanian, Prof & Head/ Aerospace**	Academic Year	:	**2024-2025 (Odd)**
Year & Branch	:	**III Aerospace**	Semester	:	**V**
Course	:	**19ASB302 – Finite Element Method for Aerospace**			

Unit: 1

* Analyse a simply supported beam subjected to a uniformly distributed load throught using Rayleigh Ritz method. Adopt one parameter trigonometric function. Evaluate the maximum deflection and BM compare with the exact solution. (AU, 2010)



Sol

From given condition one parameter trigonometric is given by

$y = a_1 \sin \frac{\pi x}{l}$ → (1) → a_1 is Ritz parameter.

we know that,

Total potential energy of the beam, $\pi = U - H$ (2)

Where, U = Strain energy

H = Work done by external force

The strain energy, U, of the beam due to bending is given by,

$$U = \frac{EI}{2}\int_0^l \left[\frac{d^2y}{dx^2}\right]^2 dx \quad \text{—(3)}$$

$$\frac{dy}{dx} = a_1 \cos\frac{\pi x}{l} \times \frac{\pi}{l}$$

$$\frac{d^2y}{dx^2} = -\frac{a_1\pi}{l}\sin\frac{\pi x}{l}\times\frac{\pi}{l}$$

$$\frac{d^2y}{dx^2} = -\left[\frac{a_1\pi^2}{l^2}\sin\frac{\pi x}{l}\right] \quad \text{—(4)}$$

(1)

Substitute Vale in equation ③

$$U = \frac{EI}{2}\int_0^l \left[-\frac{a_1\pi^2}{l^2}\sin\frac{\pi x}{l}\right]^2 dx$$

$$U = \frac{EI}{2}\times\frac{\pi^4}{l^4}\int_0^l \left(a_1^2 \sin^2\frac{\pi x}{l}\right)dx \quad —(4)$$

$$\int_0^l \left(a_1^2 \sin^2\frac{\pi x}{l}\right)dx = a_1^2\int_0^l \frac{1}{2}\left[1-\cos\frac{2\pi x}{l}\right]dx$$

$$\therefore \left[\sin^2 x = \frac{1-\cos 2x}{2}\right]$$

$$= \frac{a_1^2}{2}\int_0^l \left[1-\cos\frac{2\pi x}{l}\right]dx$$

$$= \frac{a_1^2}{2}\left[\int_0^l dx - \int_0^l \cos\frac{2\pi x}{l}dx\right]$$

$$= \frac{a_1^2}{2}\left[(x)_0^l - \left[\frac{\sin\frac{2\pi x}{l}}{2\pi/l}\right]_0^l\right]$$

$$= \frac{a_1^2}{2}\left[l - \frac{1}{2\pi}(0-0)\right] = \frac{a_1^2 l}{2}$$

$$\int_0^l \left[a_1^2 \sin^2\frac{\pi x}{l}\right]dx = \frac{a_1^2 l}{2}$$

Substitute about value in equation (4), we get

$$U = \frac{EI}{2}\times\frac{\pi^4}{l^4}\left[\frac{a_1^2 l}{2}\right]$$

$$U = \frac{EI}{2}\times\frac{\pi^4}{l^3}\left[\frac{a_1^2}{2}\right]$$

②

We know that,

Work done, $H = \int_0^l w y \, dx$

$= \int_0^l w \left(a_2 \sin \frac{\pi x}{l}\right) dx$ by external force

$H = W \int_0^l \left[a_1 \sin \frac{\pi x}{l}\right] dx$

$= W \left[a_1 \left[\frac{-\cos \frac{\pi x}{l}}{\pi / l} \right]_0^l \right]$

$= W \left[-a_1 \frac{l}{\pi} \left[\cos \frac{\pi x}{l}\right]_0^l \right]$

$= W \left[-a_1 \frac{l}{\pi} [-1-1] \right]$

$= W \left[\frac{2a_1 l}{\pi} \right]$

$H = \frac{2Wl}{\pi} [a_1]$ ---- (6)

Substitute (5) and (6) values in equation (2)

$\Pi = U - H$

$= \frac{EI}{2} \times \frac{\pi^4}{l^3} \left[\frac{a_1^2 l}{2}\right] - \frac{2wl}{\pi}[a_1]$

$\Pi = \frac{EI\pi^4}{2l^3}[a_1^2] - \frac{2Wl}{\pi}[a_1]$

For stationary value of Π, the following condition must be satisfied.

$\frac{\delta \Pi}{\delta a_1} = \frac{EI\pi^4}{4l^3} 2a_1 - \frac{2wl}{\pi} = 0 \quad \left[\frac{\delta \Pi}{\delta a_1} = 0\right]$

(3)

$$\frac{EI\pi^4}{4l^2} 2a_1 = \frac{2wl}{\pi}$$

$$a_1 = \frac{4wl^4}{EI\pi^5}$$

Substitute Value a_1, (1)

$$y = a_1 \sin\frac{\pi x}{l}$$

$$y = \frac{4wl^4}{EI\pi^5}\sin\frac{\pi x}{l} \quad \text{—— (4)}$$

we know that, maximum deflection occurs at $x = l/2$

Substitute $x = l/2$ in equation

$$y_{max} = \frac{4wl^4}{EI\pi^5}\sin\frac{\pi \times l/2}{l}$$

$$= \frac{4wl^4}{EI\pi^5}\sin\frac{\pi}{2} \qquad \because \sin\frac{\pi}{2} = 1$$

$$= \frac{4wl^2}{EI\pi^5} = 0.01307\frac{wl^4}{EI}$$

we know that, simply supported beam subjected to uniformly distributed load, maximum deflection is

$$y_{max} = \frac{5}{384}\frac{wl^4}{EI}$$

i.e, $y_{max} = 0.01307\frac{wl^4}{EI}$

Hence, exact solution and Raleigh-Ritz solution are compared.

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Unit: 1

Bending moment at midspan

Bending moment, $M = EI\frac{d^2y}{dx^2}$

We know equation (A)

$$\frac{d^2y}{dx^2} = -\left[\frac{a_1\pi^2}{l^2}\sin\frac{\pi x}{l}\right]$$

Substitute a_1 value,

$$\frac{d^2y}{dx^2} = -\left[\frac{4Wl^4}{EI\pi^5} \times \frac{\pi^2}{l^2}\sin\frac{\pi x}{l}\right]$$

Maximum bending occurs at $x = l/2$

$$\frac{d^2y}{dx^2} = -\left[\frac{4wl^4}{EI\pi^5} \times \frac{\pi^2}{l^2}\sin\frac{\pi\, l/2}{l}\right]$$

$$\frac{d^2y}{dx^2} = -\left[\frac{4wl^4}{EI\pi^5} \times \frac{\pi^2}{l^2}\sin\frac{\pi}{2}\right] \quad \left(\because \sin\frac{\pi}{2} = 1\right)$$

$$\frac{d^2y}{dx^2} = -\left[\frac{4Wl^4}{EI\pi^5} \times \frac{\pi^2}{l^2} \times 1\right]$$

$$\frac{d^2y}{dx^2} = -\frac{4Wl^2}{EI\pi^3}$$

$$\frac{d^2y}{dx^2} = -0.1294\frac{wl^2}{EI}$$

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Substitute $\frac{d^2y}{dx^2}$ value in bending moment equation

$$M_{mid} = EI \times (-0.129)\frac{wl^2}{EI}$$

$$= -0.129wl^2$$ [Negative sign indicates downward load]

we know that, simply supported beam maximum bending moment is

$$M_{mid} = \frac{wl^2}{8} = 0.125wl^2$$

Hence, exact solution and Rayleigh-Ritz solution are compared.