SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
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COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF AEROSPACE ENGINEERING

Faculty Name	:	**Dr.M.Subramanian, Prof & Head/ Aerospace**	Academic Year	:	**2024-2025 (Odd)**
Year & Branch	:	**III Aerospace**	Semester	:	**V**
Course	:	**19ASB302 – Finite Element Method for Aerospace**			

Unit: 1

Governing Equation [mathematical Model]

Definition

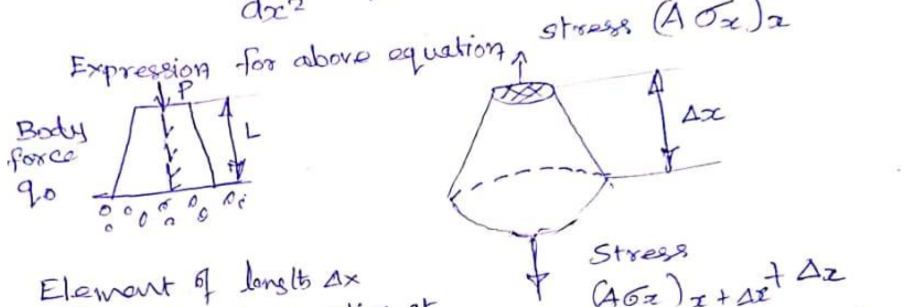
It can be broadly defined as a set of equations that express the essential features of a physical system in terms of variables that describe the system.

Examples of governing equation

* A Solid mechanics problems

$$AE\frac{d^2u}{dx^2} + q_0 = 0$$

Expression for above equation



Element of length Δx with axial forces acting at both ends of the element,

σ_x - Stress - in x direction, q_0 denotes body force, measured per unit volume $(N m^{-3})$

$(A\sigma_x)_x$ - is the net tensile force on volume element at x.

$(A\sigma_x)_{x+\Delta x}$ is the net tensile force at $x+\Delta x$

Then setting the sum of the vertical forces to zero.

$$-(A\sigma_x) + (A\sigma_x)_{x+\Delta x} + q_0 A \Delta x = 0$$

Dividing throughout by Δx and taking limit $\Delta x \to 0$, we obtain

$$-\frac{(A\sigma_x)_x}{\Delta x} + \frac{(A\sigma_x)_{x+\Delta x}}{\Delta x} + \frac{q_0 A \Delta x}{\Delta x} = 0$$

$$\lim_{\Delta x \to 0}\left[\frac{(A\sigma_x)_{x+\Delta x} - (A\sigma_x)_x}{\Delta x}\right] + q_0 A = 0$$

$$\frac{d}{dx}(A\sigma_x) + Aq_0 = 0 \quad -①$$

The stress σ_x can be related to axial displacement using Hooke's law

$$\sigma_x = E\varepsilon_x \qquad \varepsilon_x = \frac{du}{dx} \quad -②$$

E - young's modulus (N/m^2)

u - Axial displacement (m)

ε_x - Axial strain

Substitute equation (2) in equation ①, we have

$$\frac{d}{dx}\left[EA\frac{du}{dx}\right] + Aq_0 = 0 \qquad 0 < x < L$$

$$\boxed{EA\frac{d^2u}{dx^2} + Aq_0 = 0}$$

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Steady - State problems

is that the response of the System does not change with time. Thus, the State variables describing the response of the system under consideration can be obtained from the solution of a set of equation that do not involve time as a variable.

Ex: Elastic Spring System
Heat transfer system
Hydraulic networks

Propagation problems. (Dynamic problem)

is that the response of the System under consideration changes with time. For the analysis of a system, in principle the Same procedures as in the analysis of a steady-state problem are employed, but now the state variables and element equilibrium relation depend on time. The objective of the analysis is to calculate the state variable for all time.

Prepared: Dr. M. Subramanian/Professor & Head Aerospace Engineering

Eigenvalue problem

Steady-state and propagation problems we implied the existence of a unique solution for the response of the system.

A main characteristic of an eigenvalue problem is that there is no unique solution to the response of the system, and the objective of the analysis is to calculate the various possible solutions. Eigenvalue problems arise in both steady-state and dynamic analysis.

$$Av = \lambda B v$$

A & B are symmetric matrices

λ is a scalar, v is vector.

If λ_i and v_i satisfy, they are called an eigenvalue and an eigenvector respectively.

In steady-state analysis an eigenvalue problem of form is formulated when it is necessary to investigate the physical stability of the system under consideration.

4/7/2016 Ref: Bathe

Two categories of mathematical models are considered:

lumped-parameter models
&
Continuum-mechanics based models.

we also refere to these as "discrete-System" and "continuous System" mathematical models

In a lumbed-parameter mathematical model, the actual system response is directly described by the solution of a finite number of state variables.

Steady-state,
propagation
eignvalue problem

→ reduce the continuous-system mathematical model to discrete idealization. that can be solved in the same manner as a lumped-parameter model.

For a continuum-mechanics-based mathematical model, the formulation of the governing equation is achieved as for a lumped-parameter model, but instead of a set of algebraic equations for the unknown state variables, differential equation govern the response. The exact solution of differential equations satisfying all boundary conditions is possible only for relatively simple mathematical model, and numerical procedures must in general be employed.