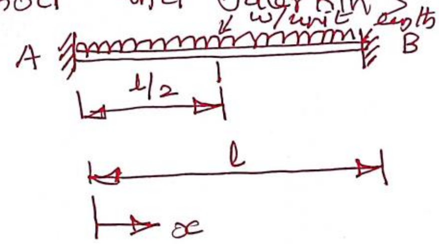
DEPARTMENT OF AEROSPACE ENGINEERING

Faculty Name	:	**Dr.M.Subramanian, Prof & Head/ Aerospace**	Academic Year	:	**2024-2025 (Odd)**
Year & Branch	:	**III Aerospace**	Semester	:	**V**
Course	:	**19ASB302 – Finite Element Method for Aerospace**			

Unit: 1

Find the deflection at the Centre of a clamped beam subjected to uniformly distributed load through its length as shown in fig. Use Collocation method and Galerkin's method.



Differential equation governed by the beam is

$$EI\frac{d^2y}{dx^2} - w = 0 \qquad 0 \le x \le l$$

Boundary condition's are,

Deflection, $y = 0$ at $x = 0$ and $x = l$

Slop, $\theta = EI\frac{dy}{dx} = 0$ at $x = 0$ & $x = l$

Solution

The trial function should satisfy the given boundary condition. Hence the trail function is considered as.

$$y = c\,(x^5 - 2lx^4 + l^2x^3)$$

$$\frac{dy}{dx} = c\,(5x^4 - 2\,(4x^3 + l^2 3x^2)$$

$$= c\,(5x^4 - 8lx^3 + 3l^2x^2)$$

①

The above two functions satisfy the given two boundary conditions.

$$\frac{d^2y}{dx^2} = C\,(20x^3 - 24lx^2 + 6l^2x)$$

$$\frac{d^2y}{dx^2} = c\,(60x^2 - 48lx + 6l^2)$$

$$\frac{d^2y}{dx^4} = C\,(120x - 48l)$$

Substituting the above value in governing differential equation, we get the residuals as

$$R_d = CEI\,(120x - 48l) - W$$

Now we apply,

(i) Collocation point method

In this method $R_d = 0$

$$\therefore CEI\,(120x - 48l) - W = 0$$

Put $x = l/2$ for maximum deflection

$$\therefore CEI\,(120 \times l/2 - 48l) = W$$

$$C = \frac{W}{EI\,12\,l}$$

Hence trail function

$$y = \frac{W}{12\,EI\,l}\,(x^5 - 2lx^4 + l^2x^2)$$

At $x = l/2$, maximum deflection

$$y_{max} = \frac{W}{12EIl}\left[\left(\frac{l}{2}\right)^2 - 2l\left(\frac{l}{2}\right)^4 + l^2\left(\frac{l}{2}\right)^2\right]$$

$$y_{max} = \frac{Wl^4}{384EI}$$ which is equal to exact solution.

ii) Galerkin's method

In this method trial function itself is considered as weighting function.

i.e., $\int_0^l W_i R_d \, dx = 0$

$$W_i = c\left[x^5 - 2lx^4 + l^2x^3\right]$$

$$R_d = cEI\left(120x - 48l\right] - W$$

Substitude W_i and R_d value in above equation

$$\Rightarrow \int_0^l \left[c(x^5 - 2lx^4 + l^2x^2) \times \left[cEI(120x - 48l) - W\right]\right]dx = 0$$

Integrating above equation and simplifying we get $c = \frac{W}{12EIl}$

Hence the trail function

$$y = \frac{W}{12EIl}\left(x^5 - 2lx^4 + l^2x^3\right)$$

At $x = l/2$ maximum deflection

$$y_{max} = \frac{W}{12EIl}\left[\left(\frac{l}{2}\right)^5 - 2l\left(\frac{l}{2}\right)^4 + l^2\left(\frac{l}{2}\right)^3\right]$$

$$y_{max} = \frac{Wl^4}{348EI}$$

(3)

A fin as shown in fig., governed by a differential equation.

$$-\frac{d}{dx}\left[KA(x)\frac{dT}{dx}\right] + hp\,(T - T_\infty) = 0$$

Boundary condition are

$T = 80°c$ at $x = 0$

$$-KA\left.\frac{dT}{dx}\right\}_{x=8} = hA\,(T - T_\infty) \text{ at } x = 8cm$$

when free end is open to the atmosphere

$$-KA\frac{dT}{dx} = 0 \text{ at } x = 8cm$$ (When free end is insulated)

Find the approximate solution when the free end

i) open to atmosphere ii) Insulated

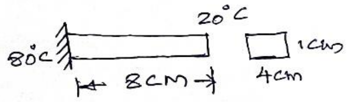
by using Collocation method. Assume four term trial solution.

Take,

$K = 3\ W/cm°C$

$h = 0.1\ W/cm^2 C$

$T_\infty = 20°C$



Data Given :

Governing equation is given by,

$$-\frac{d}{dx}\left[KA(x)\frac{dT}{dx}\right] + hp(T - T_\infty) = 0$$

Boundary conditions are,

$T = 80°c$ at $x = 0$

$$-KA\frac{dT}{dx}\Big\}_{x=8} = hA(T_\infty - 20)$$

Solution :

i) Free end is open to the atmosphere

$A(x) = 1 \times 4 = 4\,cm^2$

$P = 2 \times (4+1) = 10\,cm$

governing equation can be written as,

$$-12\left(\frac{d^2T}{dx^2}\right) + (T - 20) = 0$$

Assuming a solution

$$T(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

Applying boundary conditions 1

$a_0 = 80°c$

Applying boundary conditions 2

$$-KA\frac{dT}{dx}\Big\}_{x=8} = hA(T - 20)$$

2

$$\rightarrow 3\times 4(a_1 + 2a_2x + 3a_3x^2)_{x=8} = hA(T-20)_{x=8}$$

$$12(a_1 + 16a_2 + 192a_3) = 0.1\times 4(80 + 8a_1 + 16a_2 + 512a_3)$$

$$3a_1 + 48a_2 + 2304a_3 = 8 + 0.8a_1 + 1.6a_2 + 51.2a_2$$

$$2.2a_1 = 8 - 46.4a_2 - 2252.8a_3$$

$$a_1 = 3.64 - 21.1a_2 - 1024a_3 \quad \text{—(1)}$$

$$\therefore T(x) = 80 + (3.64 - 21.1a_2 - 1024a_2)x + a_2x^2 + a_3x^3$$

Substituting in governing equation

$$-12\frac{d^2}{dx^2}\left[80 + (3.64 - 21.1a_2 - 1024a_3)x + a_2x^2 + a_3x^3\right] + 80 + (3.64 - 21.1a_2 - 1024a_3)x + a_2x^2 + a_3x^3$$

$$\rightarrow -12\frac{d}{dx}\left[3.64 - 21.1a_2 - 1024a_3 + 2a_2x + 3a_3x^3\right] + 60 + (3.64 - 21.1a_2 - 1024a_3)x + a_2x^2 + a_3x^3 = R$$

$$\rightarrow -12(2a_2 + 6a_3x) + 60 + (3.64 - 21.1a_2 - 1024a_3)x + a_2x^2 + a_3x^3 = R(x, a_2, a_3)$$

$$\rightarrow 24a_2 - 72a_3x + 3.64x - 21.1a_2x - 1024a_3x + a_2x^2 + a_3x^3 + 60 = 0(x, a_2, a_3)$$

(3)

$$a_2(x^2-21.1x+24)+a_3(x^3-1024x-72x)$$
$$+60+3.64x = R$$

$$\Rightarrow R(x, a_2, a_3) = 3.64x+60$$
$$+a_2(x^2-21.1x-24)+a_3(x^3-1024x-72x)$$

By Collocation method

$R=0$ at $x=3$ and $x=6$

At $x=3$; $70.92+a_2(-78.3)+a_3(-3261)=0$

$78.3a_2+3261a_3 = 70.92 \quad \ldots (2)$

$\rightarrow$ At $x=6$; $R=0$

$81.84+a_2(-114.6)+a_3(-6360)=0$

$114.6a_2+6360a_3 = 81.84 \quad \ldots (3)$

Solving equation (2) and (3)

$a_2 = 1.482$

$a_3 = -0.05138$

From equation (1)

$a_1 = 3.64-21.1a_2-1024a_3$

$= 3.64-21.1\times1.482+1024\times0.0138$

$a_1 = 4.933$

Hence approximate Solution,

$$T(x) = 80-4.933x+1.482x^2-0.0138x^3$$

(ii) If the free end is insulated

Governing equation

$$-K\left\{\frac{d}{dx} A\frac{dT}{dx}\right\} + hp(T - T_\infty) = 0$$

$$-KA\frac{d^2T}{dx^2} + hp(T - T_\infty) = 0$$

Boundary condition,

$$T = 80 \text{ at } x = 0$$

$$-KA\frac{dT}{dx} = 0 \text{ at } x = l$$

Let $T(x) = a_0 + a_1x + a_3x^2 + a_3x^3$

Applying Boundary condition (1)

$$a_0 = 80^\circ C$$

Applying Boundary condition (2)

$$\left(\frac{dT}{dx}\right)_{x=8} = (a_1 + 2a_2x + \cancel{4}3a_3x^2)\Big|_{x=8} = 0$$

$$\rightarrow a_1 = (-2a_2x - 3a_3x^2)\Big|_{x=8}$$

$$a_1 = -16a_2 - 192a_3$$

$$T(x) = 80 - 16a_2x - 192a_3x + a_2x^2 + a_3x^3$$

$$T(x) = 80 + a_2(x^2 - 16x) + a_3(x^3 - 192x)$$

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Unit: 1

Substituting in the governing equation,

$$-KA\frac{d^2}{dx^2}\left[80 + a_2(x^2-16) + a_2(x^2-192)\right] + hp\left[T(x) - T_\infty\right]$$

$$-12\frac{d}{dx}\left\{a_2(2x) + 3a_3x^2\right\} + hp(T-T_\infty) = R$$

$$-12(2a_2+6a\ \ ) + 0.1\times10(80+a_2(x^2-16x) + a_3(x^3-192x)) = R$$

$$\Rightarrow R(x, a_2, a_3) = 80 + a_2(x^2-16x-24) + a_3(x^3-192x-72x)$$

$$\Rightarrow R(x, a_2, a_3) = 80 + a_2(x^2-16x-24) + a_3(x^3-264x)$$

By collocation method

$R=0$ at $x=3$

$$\Rightarrow 80 + (-63)a_2 + (-765)a_3 = 0$$

$$\Rightarrow 63a_2 + 765a_3 = 80 \rightarrow (2)$$

$R=0$ at $x=6$

$$\Rightarrow 80 + (-84)a_2 + (-1368)a_3 = 0$$

$$\Rightarrow 84a_2 + 1368a_3 = 80 \quad ---(3)$$

Solving equation (2) and (3)

$$\left.\begin{aligned} a_2 &= 2.20 \\ a_3 &= -0.076 \end{aligned}\right\}$$

From equation (1),

$$a_1 = -16a_2 - 192a_3$$
$$= -16\times2.20 + 192\times0.076$$
$$a_1 = -20.6$$

Hence the approximate solution is $T(x) = 80 - 20.6x + 2.20x^2 - 0.076x^3$ (6)