THE PRINCIPLE OF INCLUSION - EXCLUSION:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

(or) $$|A \cup B| = |A| + |B| - |A \cap B|$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D| + |B \cap C \cap D| + |A \cap C \cap D| - |A \cap B \cap C \cap D|$$

1) In a survey of 100 students, it was found that 30 studied mathematics, 54 studied statistics, 25 studied Operation Research, 1 studied all the three subjects. 20 studied mathematics and statistics, 3 studied mathematics and operations research and 15 studied statistics and Operations Research.

(i) How many students studied none of these subjects?

(ii) How many Students studied only mathematics?

Soln: Let A denote the students who studied mathematics

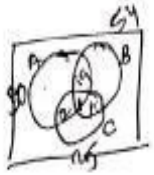
Let B denote the students who studied statistics

Let C denote the students who studied OR.

Then $|A| = 30$; $|B| = 54$; $|C| = 25$

$|A \cap B| = 20$; $|A \cap C| = 3$; $|B \cap C| = 15$

$|A \cap B \cap C| = 1$



(i) No. of students studied none of these subjects $= 100 - |A \cup B \cup C|$

$\therefore$ By principle of Inclusion - Exclusion,

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$
$$= 30 + 54 + 25 - 20 - 3 - 15 + 1$$
$$= 72$$

$\therefore$ No. of students studied none of these subjects $= 100 - 72 = 28$

(ii) No. of students studied only Maths & statistics $= n(A \cap B) - n(A \cap B \cap C)$
$= 20 - 1 = 19$

No. of students studied only Maths & OR $= n(A \cap C) - n(A \cap B \cap C)$
$= 3 - 1 = 2$

Then, No. of students studied only mathematics $= 30 - 19 - 2$
$= 9$

(2) How many positive integers not exceeding 1000 are divisible by 7 or 11?

Let A denote the set of +ve integers not exceeding 1000 that are divisible by 7.

Let B denote the set of +ve integers not exceeding 1000 that are divisible by 11.

Then $|A| = \left[\frac{1000}{7}\right] = \lceil 142.8 \rceil = 142$

$|B| = \left[\frac{1000}{11}\right] = [90.9] = 90$

$|A \cap B| = \left[\frac{1000}{7 \times 11}\right] = [12.9] = 12$.

∴ No. of +ve integers not exceeding 1000 that are divisible by either 7 or 11 is $|A \cup B|$

By principle of Inclusion-Exclusion,

$|A \cup B| = |A| + |B| - |A \cap B|$

$= 142 + 90 - 12 = 220$.

(3) Find the no. of integers between 1 & 250 that are not divisible by any of the integers 2, 3, 5 & 7.

Soln: Let A denote the integer from 1 to 250 that are divisible by 2
Let B " " " " " 3
Let C " " " " " 5
Let D " " " " " 7

$|A| = \left[\frac{250}{2}\right] = 125$

$|B| = \left[\frac{250}{3}\right] = 83$

$|C| = \left[\frac{250}{5}\right] = 50$

$|D| = \left[\frac{250}{7}\right] = 35$

Now, the no. of integers btwn 1 to 250 that are divisible by 2 & 3 $= |A \cap B| = \left[\frac{250}{2 \times 3}\right]$

$= 41$

The no. of integer divisible by 2 & 5 $= |A \cap C| = \left[\frac{250}{2 \times 5}\right]$

$= 25$

Similarly, $|A \cap D| = \left[\frac{250}{2 \times 7}\right] = 17$

$|B \cap C| = \left[\frac{250}{3 \times 5}\right] = 16$

$|B \cap D| = \left[\frac{250}{3 \times 7}\right] = 11$

$|C \cap D| = \left[\frac{250}{5 \times 7}\right] = 7$

No. of integers divisible by 2,3 & 5 $= |A \cap B \cap C|$

$= \left[\frac{250}{2 \times 3 \times 5}\right] = 8$

Similarly, $|A \cap B \cap D| = \left[\frac{250}{2 \times 3 \times 7}\right] = 5$

$|A \cap C \cap D| = \left[\frac{250}{2 \times 5 \times 7}\right] = 3$

$|B \cap C \cap D| = \left[\frac{250}{3 \times 5 \times 7}\right] = 2$

$\therefore |A \cap B \cap C \cap D| = \left[\frac{250}{2 \times 3 \times 5 \times 7}\right] = 1$

∴ No. of integers btwn. 1-250 that are divisible by integers 2, 3, 5 & 7 is $|A \cup B \cup C \cup D|$

∴ By principle Inclusion- Exclusion,

$|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D| - |A \cap B \cap C \cap D|$

$= 125 + 83 + 50 + 35 - 41 - 25 - 17 - 16 - 11 - 7 + 8 + 5 + 3 + 2 - 1$

$= 193$

∴ No. of integers btwn. 1 to 250 that are not divisible by 2, 3, 5 & 7 is $250 - |A \cup B \cup C \cup D|$

$= 250 - 193 = 57$