Unit - 1

Fundamental concepts and Energy Methods.

Determinate Structure :

The structure which can be solved with the available equilibrium equations ($\Sigma V=0, \Sigma H=0, \Sigma M=0$) is known as a determinate structure.

Ex : Simply supported beam & cantilever beam.

Indeterminate Structure :

The Structure which cannot be solved with the available equilibrium equations is known as indeterminate Structures.

Ex : Continuous beam, Propped cantilever beam & fixed beams.

Consistent Deformation method for pin-jointed Frames — Procedure.

Step - 1 : Calculate the indeterminacy.

$$I = I_i + I_e$$

Step - 2 : Calculate (F) forces of the members using method of joints for the actual external loads.

Step - 3 : Calculate K - forces of the members using method of joints by removing all external loads and apply unit force along the redundant.

Step - 4 : Calculate the redundant force using the formula.

$$T = \frac{-\Sigma \frac{FKl}{AE}}{\Sigma \frac{K^2 l}{AE}}$$

SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

ACADEMIC YEAR 2024-25

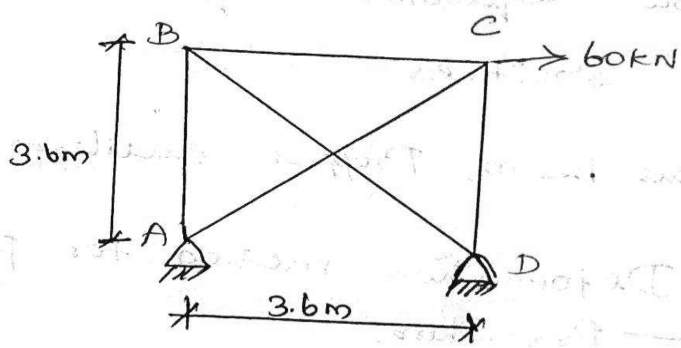
Department of Civil Engineering (SA I)

Step - 5: Calculate forces in other members, using formula.

$$P = F + kT$$

Step - 6: Do all the calculations in a tabular form.

1. Determine the forces in the members of the truss shown below by consistent deformation method. AE is constant for all the members.



Solution:

Step - 1: Calculation of Indeterminacy:

$$I_i = m - (2j - 3)$$
$$= 5 - (2\times4) + 3$$
$$= 0$$
$$I_e = r - 3 = 4 - 3 = 1$$

Hence the structure is externally indeterminate to first degree. Replace the hinged support at D by a roller support so that the horizontal force is treated as the redundant.

Step - 2 : Calculation of F-forces using method of Joints:

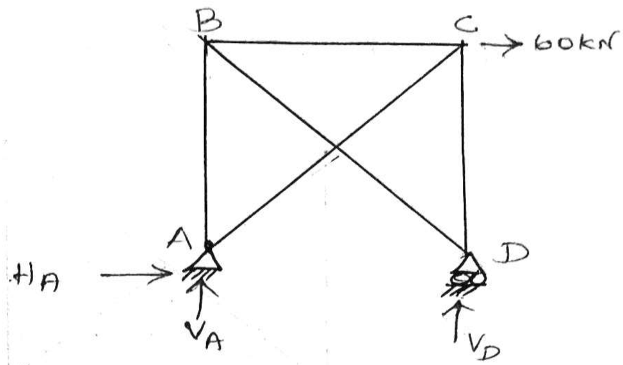
At Joint A :

$\Sigma M_A = 0 \Rightarrow$

$-V_D \times 3.6 + 60 \times 3.6 = 0$

$V_D = 60\ kN\ (\uparrow)$

$V_A = 60\ kN\ (\downarrow)$

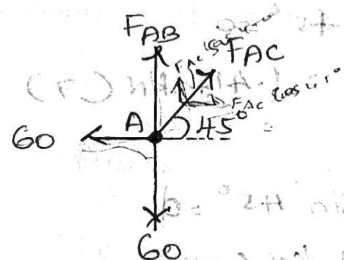


$\Sigma H = 0,$

$+H_A \mp 60 = 0$

$H_A = 60\ kN\ (\leftarrow)$

Joint A :



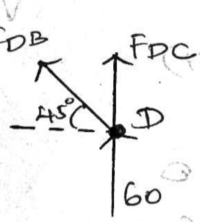
$\Sigma H = 0 \Rightarrow -60 + F_{AC} \cos 45° = 0$

$F_{AC} = 84.85\ kN\ (T)$

$\Sigma V = 0 \Rightarrow F_{AB} - 60 + F_{AC} \sin 45° = 0$

$F_{AB} = 0$

Joint D :



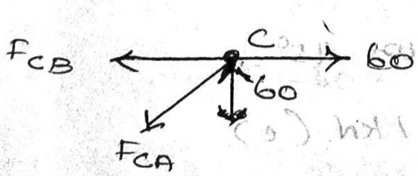
$\Sigma H = 0 \Rightarrow -F_{DB} \cos 45° = 0$

$F_{DB} = 0$

$\Sigma V = 0 \Rightarrow F_{DC} + 60 + F_{DB} \sin 45° = 0$

$F_{DC} = 60\ kN\ (C)$

Joint C :



$\Sigma H = 0 \Rightarrow -F_{CB} - F_{CA} \cos 45° + 60 = 0$

$F_{CB} = 0$

SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

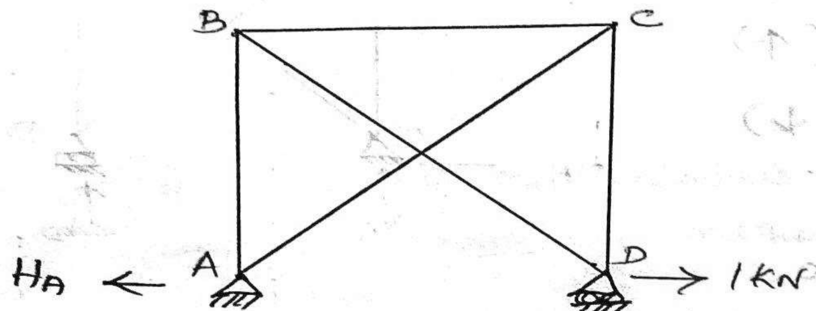
COIMBATORE-641 035, TAMIL NADU

ACADEMIC YEAR 2024-25

Department of Civil Engineering (SA I)

Step - 3 : Calculation of k - forces :

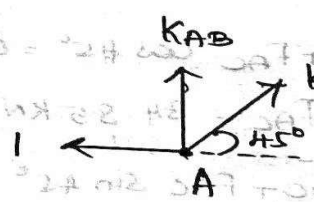
Remove all the external loads and apply unit force at D and use method of joints.



$\Sigma H = 0$

$H_A = 1$ kN

Joint A :



$\Sigma H = 0$

$-1 + K_{AC} \cos 45° = 0$

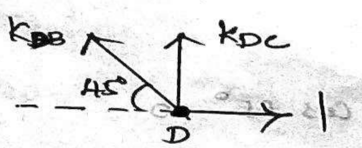
$K_{AC} = 1.414$ kN (T)

$\Sigma V = 0$

$K_{AB} + K_{AC} \sin 45° = 0$

$K_{AB} = 1$ kN (C)

Joint D :



$\Sigma H = 0$

$1 - K_{DB} \cos 45° = 0$

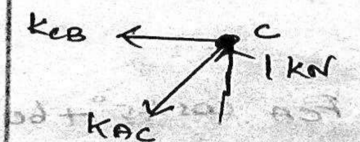
$K_{DB} = 1.414$ kN (T)

$\Sigma V = 0$

$K_{DC} - K_{DB} \sin 45° = 0$

$K_{DC} = 1$ kN (C)

Joint C :



$\Sigma H = 0,$

$-K_{CB} - 1.414 \cos 45° = 0$

$K_{CB} = 1$ kN (C)

Step-4 : Tabulation :

S.No	Member	F (kN)	k (kN)	l	Fkl	k^2l	P = F + k.T
1.	AB	0	-1	3.6	0	3.6	26.53
2.	BC	0	-1	3.6	0	3.6	26.53
3.	CD	-60	-1	3.6	216	3.6	-33.47
4.	AC	84.85	1.414	5.09	610.69	10.18	47.34
5.	BD	0	1.414	5.09	0	10.18	-37.51
					826.69	31.16	

Hence horizontal reaction @ D,

$$T = \frac{-\Sigma Fkl/AE}{\Sigma k^2 l/AE}$$

$$= \frac{-826.69}{31.16}$$

$$= -26.53 \text{ kN}$$

$$H_D = 26.53 \text{ kN} (\leftarrow)$$

$$P_{AB} = 0 + (-1) \times (-26.53) = 26.53 \text{ kN}$$

$$P_{BC} = 0 + (-1)(-26.53) = 26.53 \text{ kN}$$

$$P_{CD} = -60 + (-1)(-26.53) = -33.47 \text{ kN}$$

$$P_{AC} = 84.85 + (1.414 \times -26.53) = 47.34 \text{ kN}$$

$$P_{BD} = 0 + (1.414 \times -26.53) = -37.51 \text{ kN}$$

SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

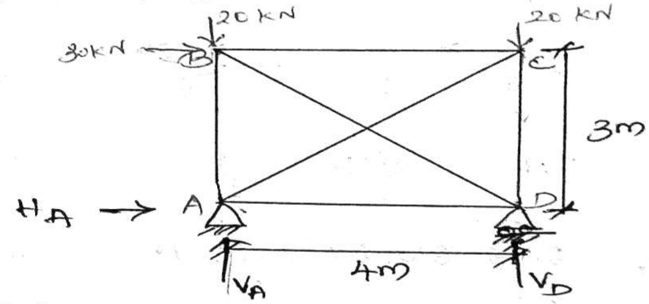
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

ACADEMIC YEAR 2024-25

Department of Civil Engineering (SA I)

2. Find the forces in all the members of the frame shown below in which the c/s Area of vertical members are 300 mm² each, and those of all other members are 2200 mm². $E = 2 \times 10^5$ N/mm²



Soln :

Step-1: Calculation of Indeterminacy :

$$I_i = m - 2j + 3$$
$$= 6 - (2\times4) + 3$$
$$= 1$$
$$I_e = r - 3 = 3 - 3 = 0$$
$$I = I_i + I_e = 1$$

Here the frame is internally indeterminate to degree 1. Hence member AC is removed to make the structure determinate.

Step-2: Calculation of F-forces :

$\Sigma M_A = 0,\quad -R_D \times 4 + (20\times4) + (30\times3) = 0$

$$R_D = 42.5 \text{ kN}$$

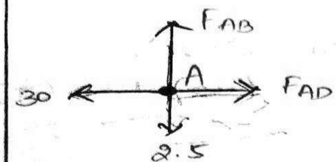
$\Sigma V = 0,\quad R_A + R_D = 40$

$$R_A + 42.5 = 40$$
$$R_A = -2.5 \text{ kN}$$
$$R_A = 2.5 \text{ kN} (\downarrow)$$

$\Sigma H = 0,\quad 30 + H_A = 0$

$$H_A = -30 \text{ kN}$$
$$H_A = 30 \text{ kN} (\leftarrow)$$

Joint A :



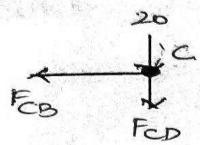
$\Sigma H = 0, \quad -30 + F_{AD} = 0$

$F_{AB} = 30\ kN\ (T)$

$\Sigma V = 0, \quad F_{AB} - 2.5 = 0$

$F_{AB} = 2.5\ kN\ (T)$

Joint C :



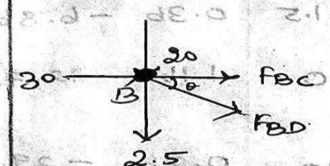
$\Sigma H = 0, \quad F_{CB} = 0$

$\Sigma V = 0, \quad -20 - F_{CD} = 0$

$F_{CD} = -20\ kN$

$F_{CD} = 20\ kN\ (C)$

Joint B :



$\tan\theta = 3/4$

$\theta = \tan^{-1}(3/4) = 36^\circ 52'$

$\Sigma H = 0, \quad 30 + F_{BD} \cos 36^\circ 52' = 0$

$F_{BD} = -37.5\ kN\ (T)$

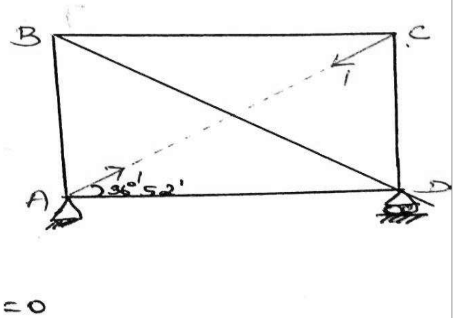
$F_{BD} = 37.5\ kN\ (C)$

$\Sigma V = 0, \quad -20 - 2.5 - F_{BD} \sin 36^\circ 52' = 0$

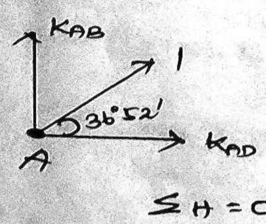
$F_{BD} = 37.5\ kN\ (C)$

Step 3: Calculation of k - forces :

Remove all the external forces & apply unit force along AC.



Joint A :



$\Sigma H = 0, \quad K_{AD} + 1 \cos 36^\circ 52' = 0$

$K_{AD} = 0.8\ kN\ (C)$

$\Sigma V = 0, \quad K_{AB} + 1 \sin 36^\circ 52' = 0$

$K_{AB} = 0.6\ kN\ (C)$

SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

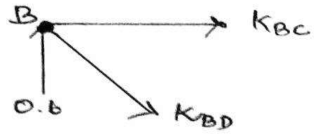
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

ACADEMIC YEAR 2024-25

Department of Civil Engineering (SA I)

Joint B :



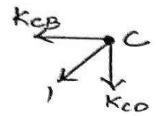
$\Sigma H = 0, \quad K_{BC} + K_{BD} \cos 36°52' = 0$

$K_{BC} = 0.8 \text{ kN (C)}$

$\Sigma V = 0, \quad 0.6 - K_{BD} \sin 36°52' = 0$

$K_{BD} = 1 \text{ kN (T)}$

Joint C :



$\Sigma V = 0, \quad -K_{CD} - 1 \sin 36°52' = 0$

$K_{CD} = 0.6 \text{ kN (C)}$

Step - 4 : Tabulation :

Sl.No	Member	F (kN)	K (kN)	l (mm)	A (mm²)	$\frac{Fkl}{A}$	$\frac{K^2 l}{A}$	P = F+kT
1.	AB	2.5	-0.6	3000	3000	-1.5	0.36	-6.866
2.	BC	0	-0.8	4000	2200	0	1.16	-12.488
3.	CD	-20	-0.6	3000	3000	12	0.36	-29.366
4.	DA	30	-0.8	4000	2200	-43.63	1.16	17.512
5.	BD	-37.5	-1.0	5000	2200	-85.23	2.27	-21.89
6.	AC	0	1.0	5000	2200	0	2.27	15.61
						-118.36	7.58	

Force in the redundant AC is

$$T = \frac{-\Sigma Fkl/AE}{\Sigma k^2 l/AE}$$

$$= \frac{-(-118.36)}{7.58}$$

$$T = 15.61 \text{ kN (T)}$$

$P_{AB} = -6.866 \text{ kN}$ $\quad P_{DA} = 17.512 \text{ kN}$

$P_{BC} = -12.488 \text{ kN}$ $\quad P_{BD} = -21.89 \text{ kN}$

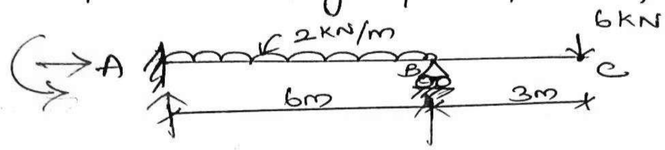
$P_{CD} = -29.366 \text{ kN}$ $\quad P_{AC} = 15.61 \text{ kN}$

SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)
COIMBATORE-641 035, TAMIL NADU
ACADEMIC YEAR 2024-25
Department of Civil Engineering (SA I)

Principle of Superposition:

The Principle of super position simply states that on a linear elastic structure, the combined effect of several loads acting simultaneously is equal to the algebric sum of the effects of each load acting individually.

3. Solve the problem by principle of superposition.



R = 4
r = 3

Soln:

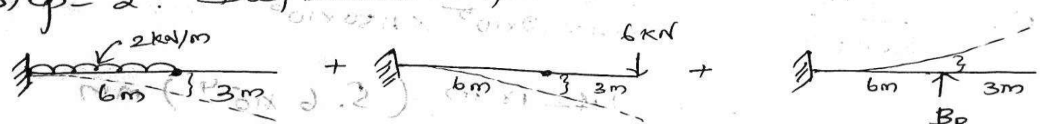
Step-1: Find indeterminacy:

$D_s = R - r$

$= 4 - 3 = 1$

Reaction at B is considered as redundant.

Step-2: Deflection of the beam:



$$\frac{-wL^4}{8EI} - \frac{PL^3}{3EI} + \frac{PL^3}{3EI} = 0$$

$$-\frac{2\times 6^4}{8EI} - \frac{6\times 9^3}{3EI} + \frac{B_R\times 6^3}{3EI} = 0$$

$$-\frac{2\times 6^4}{8} - \frac{6\times 9^3}{3} + \frac{B_R\times 6^3}{3} = 0$$

$$-324 - 1458 + 72\,B_R = 0$$

$$B_R = 24.75 \text{ kN}$$

$M_A + (24.75\times 6) - (6\times 9) + 12\times \frac{6}{2} = 0$

$\Sigma V = 0$, $24.75 - 6 - (6\times 2) + V_A = 0$

$V_A = -6.75$ kN

$\Sigma M_A = 0$,

$M_A - (6\times 2\times 6/2) + (24.75\times 6) - (6\times 9) = 0$

$M_A + 58.5 = 0$

$M_A = -58.5$ kNm

SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

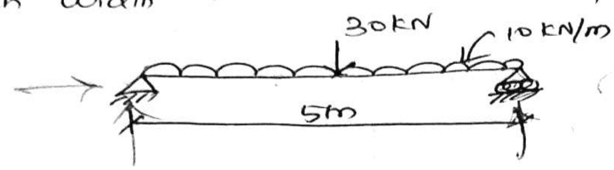
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

ACADEMIC YEAR 2024-25

Department of Civil Engineering (SA I)

4. Find deflection at centre of simply supported beam as shown below, it is a rectangular section with width 200mm and depth 350mm. $E = 2\times10^5$



$R = 3$
$r = 3$

$= 2\times10^5$

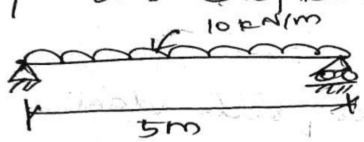
Soln :

Step - 1 : Find indeterminacy :

$D_S = R - r$
$= 3 - 3 = 0$

Statically determinate structure.

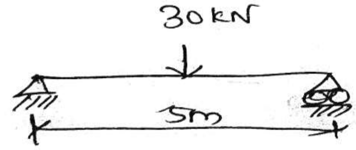
Step - 2 : Deflection of the beam :



$I = \frac{bd^3}{12} = \frac{200\times350^3}{12}$
$= 450\times10^6$

$$\delta_1 = \frac{5wL^4}{384EI} = \frac{5\times10\times5000^4}{384\times2\times10^5\times450\times10^6} \quad \frac{N/mm \times mm^4}{N/mm^2\times mm^4}$$

$= 0.9042$ mm (5.6×10^{-4}) mm



$$\delta_2 = \frac{WL^3}{48EI} = \frac{30\times1000\times5000^3}{48\times2\times10^5\times450\times10^6} \quad \frac{N\times mm^3}{N/mm^2\times mm^4} = mm$$

$= 0.8681$ mm

Total deflection = 0.9042 + 1.7723 mm

<u>Degree of Freedom:</u>

The Number of Independent joint Displacement in a structure is termed as Degree of Freedom.

<u>Kinematic Indeterminacy:</u> Associated with DOF of structure.

Methods of Structural Analysis

1. Force Method:
   * Used to analyse the indeterminate structures.
   * Used to compute internal forces and reactions.
   * Frames of single storey and uncommon geometry such as gabled frame.

2. Displacement Method:
   * Unknown displacements are taken as interms of load - displacement relationship
   * By solving equillibrium equations the unknown displacement and loads are determined.

2.1 Slope Deflection Method:
   * Used to analyse statically determinate and indeterminate beams and frames.
   * All deformations are assumed only due to bending ~~only~~.
   * All the joints of the frames are riged. the angles between the members do not change.

2.2. Moment Distribution Method:
   * It is an approximation method used for any desired degree of accuracy.
   * In this method each joint of the structure is fixed.

2.3. Direct Stiffness method:
   * This is a matrix analysis method. equillibrium equations are formulated to a single matrix relationship.

3. Approximate Methods:

* The forces and moments determined by this method are approximate values
* Based on realistic assumptions.

3.1 Portal method:

* Used to analyse frames subjected to horizontal loads.
* Points of inflections are assumed to be located at mid height of the column.
* For beams it is at mid span.

3.2. Cantilever method:

* Used for high rise structures.
* Inflections point occurs at mid point of each girder.

3.3 Points of Inflection method:

* Used to analyse the frames subjected to vertical loads.
* Frame is reduced to statically determinate form by introducing no of points of inflection
* The loading on the frame is usually UDL.
* Axial forces are negligible.

4. Kani's Method:

* In this method unknown fixed end moments are distributed to adjacent joints.
* It is a self corrective method. The errors at any stage of iteration is corrected in subsequent steps.

Energy Methods:

1. Virtual Work principle
2. Castigliano's Principle

Principle of Virtual work:

The work done by all the forces acting on a system, during a small virtual displacement is zero.

Castigliano's first theorem:

The first partial derivative of the total internal energy (strain energy) in a structure with respect to any particular deflection component at a point is equal to the force applied at that point and in the direction corresponding to that deflection component.

$$P = \frac{\partial U}{\partial \delta}$$

Applicable for linearly or non-linearly elastic structures in which the temperature is constant and the supports are unyielding.

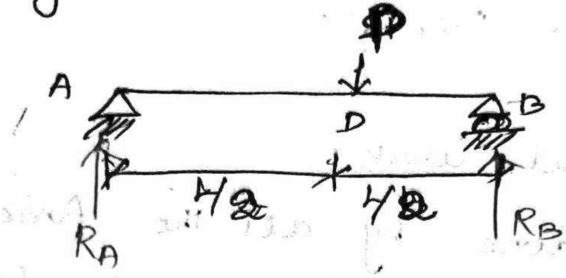
Castigliano's Second Theorem:

The first partial derivative of the total internal energy in a structure with respect to the force applied at any point is equal to the deflection at the point of application of that force in the direction of its line of action

$$\delta = \frac{\partial U}{\partial P}$$

Applicable to linearly elastic structures with constant temperature and unyielding supports.

5. Calculate the deflection of a given beam by Castigliano's Theorem.



Solution:

Step-1: Unknown Reactions:

$\Sigma V = 0, \quad R_A + R_B = P$

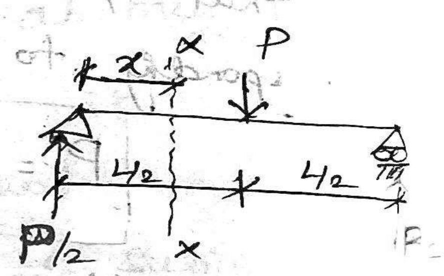
$\Sigma M_A = 0, \quad -R_B \times l + P l/2 = 0$

$R_B = P/2$

$R_A = P/2$

Step-2: Strain Energy:

Consider a section x-x at a distance x from A,



$M_x = \frac{P}{2} x$

$U = \int \frac{M^2 \, dx}{2EI}$

$= 2 \int^{l/2} \left(\frac{P}{2} x \right)^2$

By second theorem,

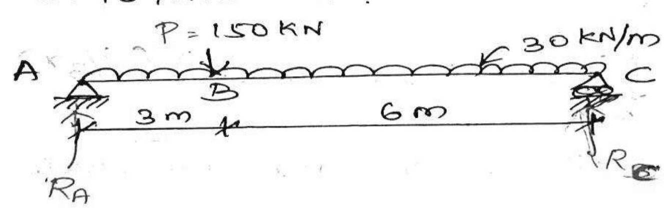
$$\delta = \frac{\partial U}{\partial W}$$

$$\delta_c = \frac{\partial\left(\frac{P^2 l^3}{96EI}\right)}{\partial P}$$

$$= \frac{2Pl^3}{96EI}$$

$$\delta_c = \frac{Pl^3}{48EI} \quad \text{(or)} \quad \frac{Wl^3}{48EI}$$

6. Determine the vertical deflection at point load 150 kN as shown in figure. Take $E = 232 \times 10^6\ kN/m^2$ and $I = 8.789 \times 10^{-4}\ m^4$.



Soln :

Step - 1 : Unknown Reactions :

$\Sigma V = 0, \quad R_A + R_B = 150 + (30 \times 9)$

$= 150 + 270$ (or) $\boxed{P + 270}$

$= 420\ kN$

$\Sigma M_A = 0, \quad 9R_C - (30 \times 9 \times 9/2) - (150 \times 3) = 0$

$9R_C - (30 \times 9 \times 9/2) - P\times 3 = 0$

$9R_C = 1665\ kN$ (or) $R_c = \left(\frac{P}{3} + 135\right) kN$

$R_C = 185\ kN$

$R_A = \left(2/3\, P + 135\right) kN$

$R_A = 235\ kN$

$-9R_c + 30 \times 9 \times \frac{9}{2} + 150 \times 3$

$-9R_c = -1215 - 450$

$= -1665$

$185\ kN$

SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

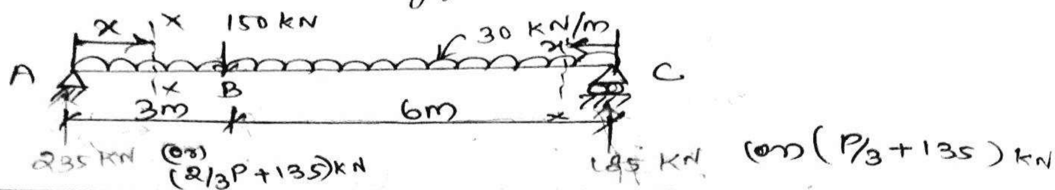
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

ACADEMIC YEAR 2024-25

Department of Civil Engineering (SA I)

Step - 2 : Strain Energy :



Segment	Origin	Limits	Moment	$\partial M/\partial P$
AB	A	0 to 3m	$235x - (30 \times x \times x/2)$ (or) $2/3 Px + 135x - 30\frac{x^2}{2}$	$2/3\,x$
CB	C	0 to 6m	$P/3\, x + 135x - 30\frac{x^2}{2}$	$1/3\,x$

$$\delta_B = \frac{1}{EI}\int_0^L M\left(\frac{\partial M}{\partial P}\right)dx$$

$$= \frac{1}{EI}\left(\int_0^3 \left(\frac{2P}{3}x + 135x - 15x^2\right) 2/3\, x \cdot dx + \int_0^6 \left(P/3\, x + 135x - 15x^2\right)\left(1/3\, x\right) dx\right)$$

Substitute $P = 150$ kN and integrate,

$$= \frac{1}{EI}\left(\int_0^3 \left(\frac{300}{3}x + 135x - 15x^2\right) 2/3\, x \cdot dx - \int_0^6 \left(\frac{150}{3}x + 135x - 15x^2\right) 1/3\, x \cdot dx\right)$$

$$= \frac{1}{EI}\left(\int_0^3 (235x - 15x^2)\, 2/3\, x \cdot dx + \int_0^6 (185x - 15x^2)\, 1/3\, x \cdot dx\right)$$

$$\delta_B = \frac{1}{EI}\left(\int_0^3 (156.67x^2 - 10x^3)\,dx + \int_0^6 (61.67x^2 - 5x^3)\,dx\right)$$

$$= \frac{1}{EI}\left[\left(\frac{156.67x^3}{3} - \frac{10x^4}{4}\right)_0^3 + \left(\frac{61.67x^3}{3} - \frac{5x^4}{4}\right)_0^6\right]$$

$$= \frac{1}{EI}\left[(1410.03 - 202.5) + (4440.24 - 1620)\right]$$

$$= \frac{4027.77}{EI}$$

$$\delta_B = \frac{4027.77}{232\times10^6 \times 8.789\times10^{-4}}$$

$$= 0.01975\,m$$

$$= 19.75\,mm$$

(or)

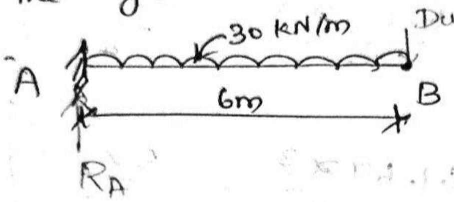
Strain Energy, $U = \int \frac{M^2 dx}{2EI}$

$$U = \frac{1}{2EI}\int_0^3 \left(2/3\,Px + 135x - \frac{30x^2}{2}\right)^2 . dx + \frac{1}{2EI}\int_0^6 \left(P/3\,x + 135x - \frac{30x^2}{2}\right) .. dx$$

$$\delta = \frac{\partial U}{\partial W} = \frac{\partial}{\partial W}\int \frac{M^2 dx}{2EI} = \int 2M . \frac{\partial M}{\partial W} . \frac{dx}{2EI}$$

$$= \frac{2}{2EI}\int M . \frac{\partial M}{\partial W} . dx$$

(7) By using Castigliano's theorem find deflection B for the given Cantilever beam.



Soln :

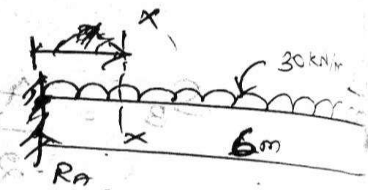
Step-1 : Unknown Reactions :

$R_A = (30 \times 6) + P$

$R_A = (180 + P)$ kN

Step-2 : Deflection under B :

$\delta_B = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial P} \cdot dx$



Consider a section x-x at x distance from A

$M_x = R_A \times x - (30 \times x \times x/2)$ $= -Px - 30x \times x/2$

$= (180 + P)x - (30x^2/2)$

Segment	Origin	Limits	Moment	$\partial M/\partial P$
AB	A	0 to 6m	$(180+P)x - \frac{30x^2}{2}$ $= 180x - 15x^2 \mp Px$	$-x$

$\delta_B = \frac{1}{EI} \int_0^6 (180x - 15x^2 \mp Px) \cdot -x \cdot dx$

$= \frac{1}{EI} \int_0^6 180x^2 (+ 15x^3 \mp Px^2)\, dx$

$$\delta_B = \frac{1}{EI}\left[\frac{180x^3}{3} + \frac{15x^4}{4} + \frac{Px^3}{3}\right]_0^6$$

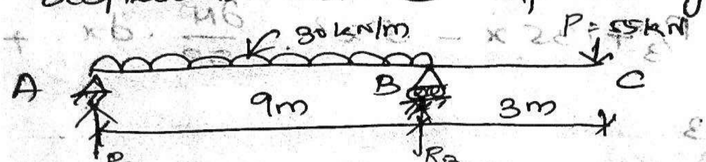
$$= \frac{1}{EI}\left[12960 + 4860 + 72P\right]$$

$$= \frac{4860}{EI} + 72P$$

Put $P = 0$ (dummy load)

$$\delta_B = \frac{4860}{EI}$$

8. Find deflection at C of the given beam.



EI = constant, $E = 200$ Gpa, $I = 830 \times 10^6$ mm⁴

Soln:

Step - 1: Unknown forces:

$R_A + R_B = P + 270$

$\Sigma M_A = 0$, $R_B \times 9 - (30 \times 9 \times 9/2) - (P \times 12) = 0$

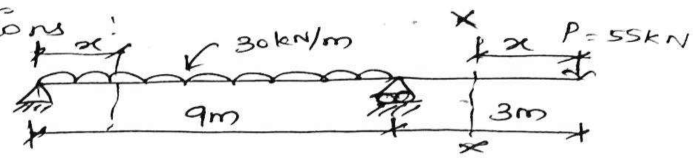
$$R_B = \frac{4P}{3} + 135$$

$\Sigma V = 0$, $R_A + R_B = P + 270$

$$R_A = (P + 270) - \left(\frac{4P}{3} + 135\right)$$

$$= P/3 + 135$$

Step - 2 : Deflections :



Segment	Origin	Limits	Moment	$\partial M/\partial P$
AB	A	0 to 9	$(\frac{P}{3}+135)x-15x^2$	$x/3$
BC	C	0 to 3	$-Px$	$-x$

$$\delta_B = \int_0^L M \cdot \frac{\partial M}{\partial P} \cdot \frac{dx}{2EI}$$

$$= \frac{1}{2EI}\int_0^9 \frac{P}{3}x + 135x - 15x^2 \cdot \frac{\partial M}{\partial P} \cdot dx \; +$$

$$\frac{1}{2EI}\int_0^3 -Px \cdot \frac{\partial M}{\partial P} \cdot dx$$

$$= \frac{1}{2EI}\left[\int_0^9 \left(\frac{P}{3}x + 135x - 15x^2\right) \frac{x}{3} \cdot dx + \int_0^3 -Px(-x) \cdot dx\right]$$

$\delta_B = 4.84$ mm

$$= \frac{1}{2EI}\left[\int_0^9 \left(\frac{Px^2}{9} + \frac{135x^2}{3} - \frac{15x^3}{3}\right) dx + \int_0^3 Px^2 \cdot dx\right]$$

$$= \frac{1}{2EI}\left[\left(\frac{Px^3}{27} + \frac{135x^3}{9} - \frac{15x^4}{9}\right)_0^9 + \left(\frac{Px^3}{3}\right)_0^3\right]$$

put P = 5

SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

ACADEMIC YEAR 2024-25

Department of Civil Engineering (SA I)

$$\delta_B = \frac{1}{2EI}\left[\frac{150 \times 9^3}{27} + \frac{135 \times 9^3}{9} - \frac{15 \times 9^4}{9} + \frac{55 \cdot \ldots}{\ldots}\right]$$

$$= \frac{1}{2EI}\left(4050 + 10935 - 10935 + 495\right)$$

$E = 200 \times 10^9 \ N/m^2$

$= 200 \times 10^6 \ kN/m^2$

$I = 830 \times 10^6 \ mm^4$

$= 830 \times 10^6 \times 10^{-12} \ m^4$

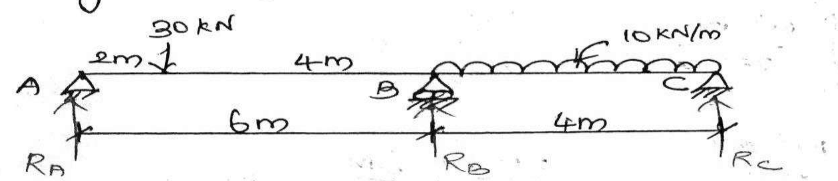
$= 830 \times 10^{-6} \ m^4$

$$\delta_B = \frac{1}{200 \times 10^6 \times 830 \times 10^{-6}}\left(4545\right)$$

$= 0.021 \ m$

$\delta_B = 21 \ mm$

9. Analyse the continuous beam loaded as shown in figure by castigliano's method.



Soln :

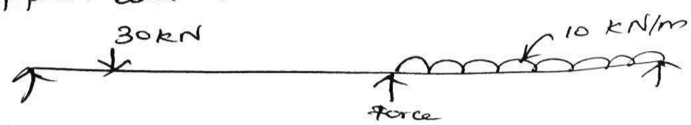
Step - 1 : Static Indeterminacy :

$D_s = R - r$ Here vertical loadings only acting

$= 3 - 2$

$D_s = 1$

R_B is treated as a redundant so remove the support and treated as force.



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

ACADEMIC YEAR 2024-25

Department of Civil Engineering (SA I)

$$10R_A = 240 + 80 - 4R_B$$

$$R_A = 32 - 0.4R_B$$

$\Sigma V = 0$

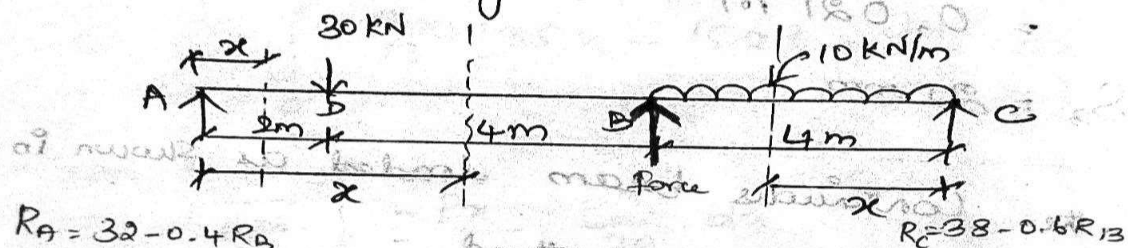
$$R_C + R_A = 30 + (10\times 4) - R_B$$

$$R_C + R_A = 70 - R_B$$

$$R_C = 70 - 32 + 0.4R_B - R_B$$

$$= 38 - 0.6R_B$$

Step - 3: Calculating Redundant:



$$U = \int_0^l \frac{M^2 dx}{2EI} \qquad \frac{\partial U}{\partial R_B} = 0$$

$$\frac{\partial}{\partial R_B}\int_0^l \frac{M^2 dx}{2EI} = \int_0^l \frac{2M}{2EI}\cdot\frac{\partial M}{\partial R_B}\cdot dx = 0$$

$$\frac{1}{EI}\int_0^l M\cdot\frac{\partial M}{\partial R_B}\cdot dx = 0$$

$$\int_0^l M\cdot\frac{\partial M}{\partial R_B}\cdot dx = 0$$

Segment	Origin	Limits	Moment	$\partial M/\partial R_B$
AD	A	0 to 2	$(32-0.4R_B)x = 32x - 0.4R_B \cdot x$	$-0.4x$
DB	A	2 to 6	$(32-0.4R_B)x - 30(x-2)$ $= 32x - 0.4R_Bx - 30x + 60$ $= 2x - 0.4R_Bx + 60$	$-0.4x$
CB	C	0 to 4	$(38-0.6R_B)x - 10 \cdot x \times x/2$ $= 38x - 0.6R_Bx - 5x^2$	$-0.6R_Bx$

$$\int_0^l M \cdot \frac{\partial M}{\partial R_B} \cdot dx = 0$$

$$\int_0^2 (32x - 0.4R_B \cdot x)(-0.4x)\,dx + \int_2^6 (2x - 0.4R_Bx + 60)(-0.4x)\,dx + \int_0^4 (38x - 0.6R_Bx - 5x^2)(-0.6x)\,dx = 0$$

$$\int_0^2 (-12.8x^2 + 0.16x^2R_B)\,dx + \int_2^6 (-0.8x^2 + 0.16x^2R_B - 24x)\,dx + \int_0^4 (-22.8x^2 + 0.36x^2R_B + 3x^3)\,dx$$

$$\left(-12.8\frac{x^3}{3} + \frac{0.16x^3}{3}R_B\right)_0^2 + \left(-0.8\frac{x^3}{3} + 0.16\frac{x^3}{3}R_B - 24\frac{x^2}{2}\right)_2^6 + \left(-22.8\frac{x^3}{3} + 0.36\frac{x^3}{3}R_B + 3\frac{x^4}{4}\right)_0^4 = 0$$

$-34.133 + 0.427 R_B + [(-57.6 + 11.52 R_B - 432) - (-2.133 + 0.427 R_B - 48)] - (486.4 + 7.68 R_B + 19.2$

$-768 + 19.2 R_B = 0$

$R_B = 40 \text{ kN}$

$R_A = 32 - 0.4 R_B = 32 - (0.4 \times 40) = 16 \text{ kN}$

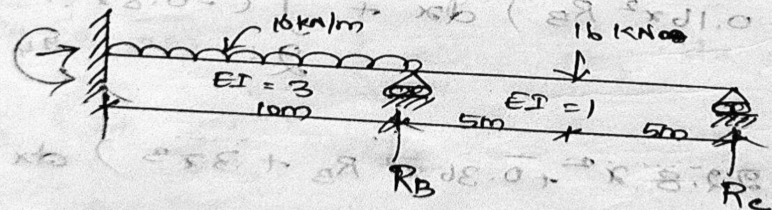
$R_C = 38 - 0.6 R_B = 38 - (0.6 \times 40) = 14 \text{ kN}$

$M_B = R_A \times 6 - 30 \times 4 = 16 \times 6 - 120 = 24 \text{ kNm}$

(or)

$M_B = R_C \times 4 - 10 \times 4 \times 4/2 = (14 \times 4) - (10 \times 8) = 24 \text{ kNm}$

10. Analyse the continuous beam loaded as shown in fig. by Castigliano's theorem.



Soln:

Step-1 : Static Indeterminacy :

$D_s = R - r$ (vertical loads only acting)

$= 4 - 2$

$= 2$

Consider R_B and R_c are redundant and remove the supports at B and C. As R_B and R_c are considered as forces.

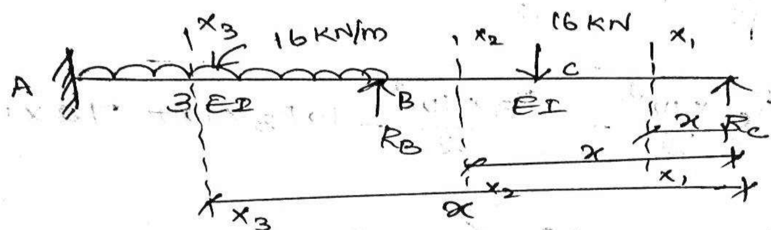
Step - 2 : Reactions :

$$U = \int \frac{M^2\, dx}{2EI}$$

$$\frac{\partial U}{\partial R_B} = \int \frac{2M}{2EI} \frac{\partial M}{\partial R_B} \cdot dx = 0$$

$$\int_0^l \frac{M}{EI} \cdot \frac{\partial M}{\partial R_B} \cdot dx = 0 \longrightarrow (1)$$

lly $\frac{\partial U}{\partial R_C} = 0 \Rightarrow \int_0^l \frac{M}{EI} \frac{\partial M}{\partial R_C} = 0 \longrightarrow (2)$



Segment	Origin	Limits	Moment	$\frac{\partial M_x}{\partial R_B}$	$\frac{\partial M_x}{\partial R_C}$	EI
CD	C	0 to 5	$R_C \times x$	0	x	1
DB	C	5 to 10	$R_C x - 16(x-5)$	0	x	1
BA	C	10 to 20	$R_C x - 16(x-5) + R_B(x-10) - \frac{16(x-10)^2}{2}$	$x-10$	x	3

$(1) \Rightarrow \frac{\partial U}{\partial R_B} = 0 \Rightarrow \int_0^l \frac{M}{EI} \cdot \frac{\partial M}{\partial R_B} \cdot dx = 0$

$$0 + 0 + \frac{1}{3}\int_{10}^{20} R_C x - 16(x-5) + R_B(x-10) - \frac{16(x-10)^2}{2} \cdot dx\,(x-10) = 0$$

$$\frac{1}{3}\int_{10}^{20} R_C x - 16x + 80 + R_B x - 10R_B - 16\frac{(x^2 + 100 - 20x)}{2} \cdot dx\,(x-10) = 0$$

$$\frac{1}{3}\int_{10}^{20}\left(R_C x - 16x + 80 + R_B x - 10R_B - 8x^2 - 800 + 160x\right)(x-10)$$

$$\frac{1}{3}\int_{10}^{20}\left(R_C x + R_B x - 10R_B + 144x - 720\right)(x-10)\,dx = 0$$

$$\frac{1}{3}\int_{10}^{20}\left(R_C x^2 + R_B x^2 - 10R_B x + 144x^2 - 720x - R_C \times 10x - 10R_B x + 100R_B - 1440x + \ldots\right)$$

$$\frac{1}{3}\left[R_C\frac{x^2}{2} + R_B\frac{x^2}{2} - 10R_B x + \frac{144x^2}{2} - 720x\right]_{10}^{20} = 0$$

$$\frac{1}{3}\left[\left(R_C\times\frac{20^2}{2} + \frac{R_B\times 20^2}{2} - 10R_B\times 20 + 72\times 20^2 - 720\times 20\right) - \left(R_C\times\frac{10^2}{2} + \frac{R_B\times 10^2}{2} - 10R_B\times 10 + 72\times 10^2 - 720\times 10\right)\right]$$

$$\frac{1}{3}\left[200R_C + 200R_B - 200R_B + 28800 - 14400 - 50R_C - 50R_B + 100R_B - 7200 + 7200\right]$$

$$\frac{1}{3}\left[200R_C - 50R_C + 100R_B - 50R_B + 28800 - 14400\right]$$

$$\frac{1}{3}\left[150R_C + 50R_B + 28800\right] = 0$$

$$\frac{150R_C}{3} + \frac{50R_B}{3} + \frac{28800}{3} = 0$$

$$50R_C + 16.67R_B + 9600 = 0$$

$$R_B + 2.5R_C = 87.99 \longrightarrow (3)$$

SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

ACADEMIC YEAR 2024-25

Department of Civil Engineering (SA I)

$$\frac{\partial U}{\partial R_C} = 0 \Rightarrow \int_0^l \frac{M}{EI} \cdot \frac{\partial M}{\partial R_C}\, dx = 0.$$

$$\frac{1}{EI}\int_0^5 R_C x \cdot x \cdot dx + \int_5^{10} R_C x - 16(x-5) \cdot x \cdot dx +$$

$$\frac{1}{3EI}\int_{10}^{20} R_C x - 16(x-5) + R_B(x-10) - 8(x-10)^2 \cdot x \cdot dx = 0$$

$$\frac{1}{EI}\int_0^5 R_C x^2 \cdot dx + \int_5^{10} R_C x \cdot x - 16x(x-5) \cdot dx +$$

$$\frac{1}{3EI}\int_{10}^{20} R_C x - 16x + 80 + R_B x - 10R_B - 8x(x-10)^2 \cdot dx = 0.$$

$$\frac{1}{EI}\int_0^5 R_C x^2 \cdot dx + \int_5^{10} (R_C x^2 - 16x^2 + 80x)\, dx +$$

$$\frac{1}{3EI}\int_{10}^{20} R_C x - 16x + 80 + R_B x - 10R_B - 8x(x^2 + 100 - 20x)\, dx = 0$$

$$\frac{1}{EI}\int_0^5 R_C x^2 \cdot dx + \int_5^{10} (R_C x^2 - 16x^2 + 80x)\, dx +$$

$$\frac{1}{3EI}\int_{10}^{20} R_C x - 16x + 80 + R_B x - 10R_B - 8x^3 - 800x + 160x^2)\, dx = 0$$

$$\frac{1}{EI}\left[\left[R_C \frac{x^3}{3}\right]_0^5 + \left[R_C \frac{x^3}{3} - \frac{16x^3}{3} + 80\frac{x^2}{2}\right]_5^{10}\right] +$$

$$\frac{1}{3EI}\left[R_C \frac{x^2}{2} - 16x^2 + 80x + R_B \frac{x^2}{2} - 10R_B x - \frac{8x^4}{4} - 800\frac{x^2}{2} + 160\frac{x^3}{3}\right)_{10}^{20} dx$$

$\frac{1}{EI}\Big(R_C \times 41.67 + R_C \times 333.33 - 5333.33 + 45..$

$41.67R_C + 666.67 - 1000\Big) +$

$\frac{1}{3EI}\Big(200R_C - 6400 + 1600 + 200R_B - 200R_B - 32000$

$16000 + 426666.67 - 50R_C - 1600 + 800 + 50R_B -$

$2000 - 40000 + 5333\Big) = 0$

$R_B + 4R_C = 92.4 \rightarrow (4)$

$R_B = 80.64\ kN$

$R_C = 2.94\ kN$

BM @ CD,

$M_x = R_C \times x$

$= 2.94 \times x$

$x = 0,\ M_C = 0,$

$x = 5,\ M_D = 14.75\ kNm$

$R_A = (16 \times 10) + 16 - R_B - R_C$

$= 160 + 16 - 80.64 - 2.94$

$R_A = 92.42\ kN$

SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

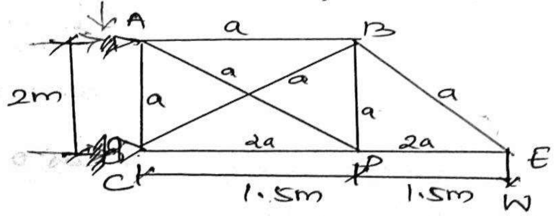
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

ACADEMIC YEAR 2024-25

Department of Civil Engineering (SA I)

11) Analyse the truss by Castigliano's theorem. The members are all of the same material.



Soln :

Step - 1 : Static indeterminacy :

$m = 8, \; j = 5 \;, \; R = 3$

$D_s = m + R - 2j$

$= (8+3) - (2 \times 5)$

$= 1$

Let the member BC be the redundant.

Replace BC and apply a force (R) ⇒ compressive

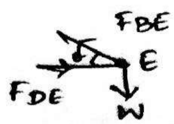
$$\frac{\partial U}{\partial R} = 0$$

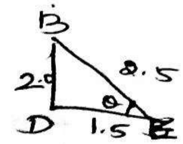
$$U = \Sigma \frac{F^2 L}{2AE}$$

$$\frac{\partial U}{\partial R} = \Sigma \frac{L}{AE} \cdot F \cdot \frac{\partial F}{\partial R} = 0$$

Member	Length (m)	Area (A)	Force F	$\partial F / \partial R$	$\frac{L}{AE} \cdot F \cdot \frac{\partial F}{\partial R} \cdot aE$	Actual Force
AB	1.5	a	0.6R+0.7W	0.6	54R + 67.5W	1.088W (T)
AC	2.0	a	0.8R	0.8	128R	0.451W (T)
AD	2.5	a	1.25W − R	−1	−312.5W + 250R	0.686W (T)
BC	2.5	a	−R	−1	250R	0.564W (C)
BD	2.0	a	−(W−0.8R)	0.8	−160W + 128R	0.549W (C)
BE	2.5	a	1.25W	0	0	1.25W (T)
CD	1.5	2a	−(1.25W −0.6R)	0.6	−67.5W + 27R	1.162W (C)
DE	1.5	2a	0.75W	0	0	0.75W (C)
					837R − 472.5W	

Joint E :





$\sin\theta = \frac{2.0}{2.5}$

$\cos\theta = \frac{1.5}{2.5}$

$\Sigma V = 0,$

$F_{BE} \sin\theta = W$

$F_{BE} = \frac{W}{\sin\theta} = \frac{W}{4/5} = 1.25W$ (or) $\frac{5W}{4}$

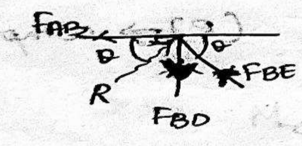
$\Sigma H = 0,$

$F_{DE} = F_{BE} \cos\theta$

$= 1.25W \times 3/5$

$F_{DE} = 0.75W$ (compressive)

Joint B:



$\Sigma V = 0,$

$F_{BD} - R\sin\theta - F_{BE}\sin\theta = 0$

$F_{BD} = (5/4\, W \times 4/5) + (R \times 4/5)$

$F_{BD} = W + 0.8R$ (compressive)

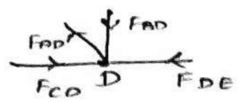
$\Sigma H = 0,$

$F_{AB} - F_{BE}\cos\theta - R\cos\theta = 0$

$F_{AB} = 3/5 R + 5/4\, W \times 3/5$

$F_{AB} = (0.6R + 0.75W)$ Tension.

Joint D :



$\Sigma V = 0$, $F_{AD} \sin\theta = F_{BD}$

$F_{AD} = (W - 0.8R)\, 5/4$

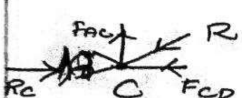
$F_{AD} = 1.25W - R$ (Tension)

$\Sigma H = 0$, $F_{CD} - F_{DE} - F_{AD} \cos\theta = 0$

$F_{CD} = (1.25W - R)\, 3/5 + 0.75W$

$= 1.5W - 0.6R$ (compressive)

Joint C :



$\Sigma V = 0$, $F_{AC} - R \cdot \sin\theta = 0$,

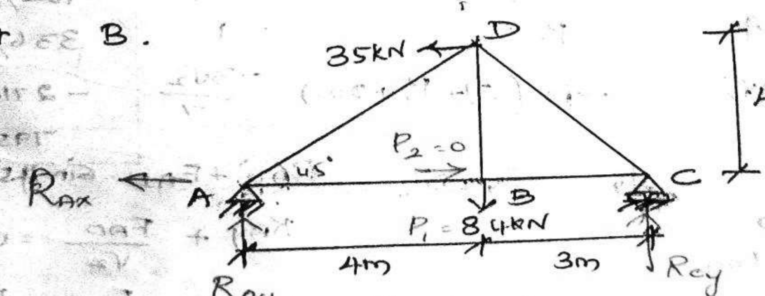
$F_{AC} = R \times 4/5 = 0.8R$

(tension)

$\frac{\partial U}{\partial R} = 0$, $837R - 472.5W = 0$

$R = 0.564W$

(12) Use Castigliano's second theorem to determine the horizontal and vertical components of the deflection at point B.



$\sqrt{4^2+4^2} = 4\sqrt{2}$

$m = 5$, $R = 3$

$j = 4$

EA = constant, $E = 200$ Gpa

$A = 1200 mm^2$

Soln :

Static Indeterminancy : $D_s = (m+R) - 2j$

$= (5+3) - (2 \times 4) = 0$

SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

ACADEMIC YEAR 2024-25

Department of Civil Engineering (SA I)

Soln :

Unknown Reactions :

$\Sigma M_A = 0$, R_{cy}

$R_{cy} \times 7 + 35(4) - (P_1 \times 4) = 0$

$R_{cy} = \frac{4P_1}{7} - 20$

$\Sigma F_V = 0$,

$R_{cy} + R_{Ay} - P_1 = 0$

$R_{Ay} = P_1 - R_{cy}$

$= P_1 - \left(\frac{4P_1}{7} - 20\right)$

$R_{Ay} = \frac{3P_1}{7} + 20$

$\Sigma H = 0$,

$-R_{Ax} + P_2 - 35 = 0$

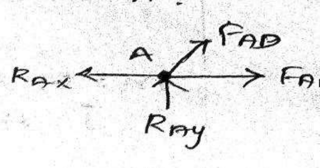
$R_{Ax} = P_2 - 35$

Deflection at B :

$P_1 = 84$, $P_2 = $

Member	Length (m)	Force	$\partial F/\partial P_1$	$\frac{FL}{AE} \partial F/\partial P_1 = L$
AB	4	$P_2 + 3/7 P_1 - 15$	3/7	36/AE
BC	3	$3/7 P_1 - 15$	3/7	27/AE
CD	5	$5/7 P_1 - 25$	5/7	125/AE
BD	4	P_1	1	336/AE
AD	$4\sqrt{2}$	$-\sqrt{2}(3/7 P_1 + 20)$	$-\frac{3\sqrt{2}}{7}$	$-271.53/AE$
				795.53/AE

Joint A :



$\Sigma V = 0$, $R_{Ay} + F_{AD} \sin 45° = 0$

$R_{Ay} + \frac{F_{AD}}{\sqrt{2}} = 0$

$\frac{F_{AD}}{\sqrt{2}} = -R_{Ay}$

$F_{AD} = -R_{Ay}$ (

$F_{AD} = -\sqrt{2}$ (3/7

SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)
COIMBATORE-641 035, TAMIL NADU
ACADEMIC YEAR 2024-25
Department of Civil Engineering (SA I)

$\Sigma H = 0, \quad P_2 - 35 = R_{Ax}$

$-R_{Ax} + (F_{AD} \times \frac{1}{\sqrt{2}}) + F_{AB} = 0$

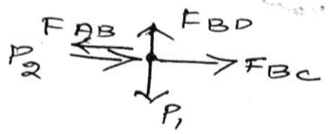
$F_{AB} = R_{Ax} - F_{AD} \times \frac{1}{\sqrt{2}}$

$= (P_2 - 35) - (-\sqrt{2})(\frac{3}{7}P_1 + 20) \times \frac{1}{\sqrt{2}}$

$= P_2 - 35 + \frac{3}{7}P_1 + 20$

$= P_2 + \frac{3}{7}P_1 - 15$

Joint B:



$\Sigma V = 0,$

$F_{BD} - P_1 = 0$

$F_{BD} = P_1$

$\Sigma H = 0,$

$F_{BC} + P_2 - F_{AB} = 0$

$F_{BC} = F_{AB} - P_2$

$= (P_2 + \frac{3}{7}P_1 - 15) - P_2$

$= \frac{3}{7}P_1 - 15$

Joint C:

$\Sigma V = 0,$

$R_{cy} - F_{CD} \sin\theta = 0 \qquad \sin\theta = \frac{op}{hy} = 4/5$

$R_{cy} - F_{CD} \times 4/5 = 0$

$\frac{4}{5} F_{CD} = R_{cy}$

$F_{CD} = \frac{5}{4} \times (\frac{4}{7}P_1 - 20)$

$= \frac{5}{7}P_1 - 25$

From table,

$\Delta V_B = \frac{795.33 \times 1000 \times 1000}{1200 \times 200 \times 10^3} \frac{N.mm}{N/mm^2 \times mm^2}$

$= 3.315\ mm\ (\downarrow)$

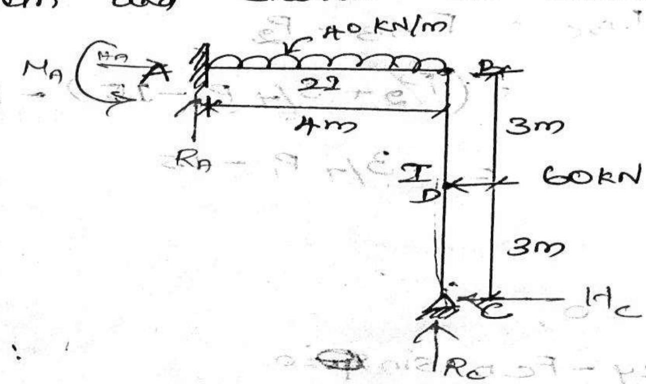
Member	Length (m)	Force	$\partial F/\partial P_2$
AB	4	$P_2 + 3/4 P_1 - 15$	1
BC	3	$3/4 P_1 - 15$	0
CD	5	$5/4 P_1 - 25$	0
BD	4	P_1	0
AD	$4\sqrt{2}$	$-\sqrt{2}(3/4 P_1 + 20)$	0

$$\Delta_{HB} = \frac{84 \times 1000 \times 1000}{1200 \times 200 \times 10^3} \quad \frac{N \cdot mm}{mm^2 \times N/mm^2}$$

$$\Delta_{HB} = 0.35 \text{ mm } (\rightarrow)$$

$$\Delta_B = \sqrt{\Delta_H^2 + \Delta_V^2}$$

13. Analyse the frame as shown in figure by Castigliano theorem and sketch the bending moment diagram.



3m + R - 3j
3(3) +

Soln :

Static Indeterminacy : m = 3, R = 5, j = 4

$D_S = 3m + R - 3j$
$= (3 \times 3) + 5 - (3 \times 4)$
$= 14 - 12$
$= 2$

R_C and H_C are considered as redundant. Remove the support to make given structure a determinate.

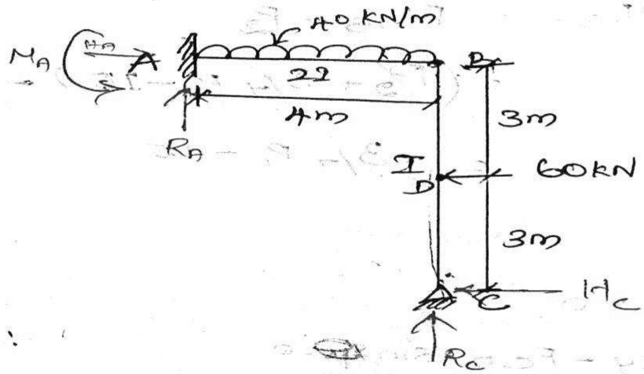
Member	Length (m)	Force	$\partial P / \partial P_2$	Δ_B
AB	4	$P_2 + 3/4 P_1 - 15$	1	$84/\ldots$
BC	3	$3/4 P_1 - 15$	0	0
CD	5	$5/4 P_1 - 25$	0	0
BD	4	P_1	0	0
AD	$4\sqrt{2}$	$-\sqrt{2}(3/4 P_1 + 20)$	0	0
				$84/\ldots$

$$\Delta_{HB} = \frac{84 \times 1000 \times 1000}{1200 \times 200 \times 10^3} \quad \frac{N \cdot mm}{mm^2 \times N/mm^2}$$

$$\Delta_{HB} = 0.35 \text{ mm } (\rightarrow)$$

$$\Delta_B = \sqrt{\Delta_H^2 + \Delta_V^2}$$

13. Analyse the frame as shown in figure by Castigliano theorem and sketch the bending moment diagram.



$3m + R - 3j$
$3(3) +$

Soln :

Static Indeterminacy : $m = 3$, $R = 5$, $j = 4$

$$D_s = 3m + R - 3j$$
$$= (3 \times 3) + 5 - (3 \times 4)$$
$$= 14 - 12$$
$$= 2$$

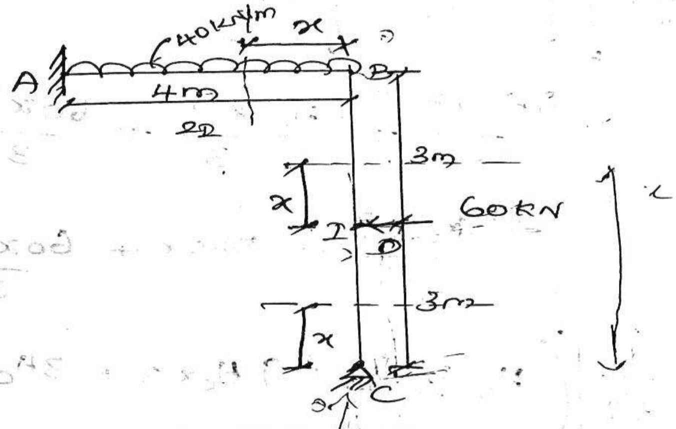
R_c and H_c are considered as redundant. Remove the support to make given structure a determinate.

Redundant Calculation :

$$U = \int \frac{M^2}{2EI} \cdot dx$$

$$\frac{\partial U}{\partial H_c} = 0, \Rightarrow \frac{\partial U}{\partial H_c} = \frac{1}{EI}\int M \frac{\partial M}{\partial H_c} \cdot dx = 0 \rightarrow (1)$$

$$\frac{\partial U}{\partial R_c} = 0, \Rightarrow \frac{\partial U}{\partial R_c} = \frac{1}{EI}\int M \cdot \frac{\partial M}{\partial R_c} \cdot dx = 0 \rightarrow (2)$$



Segment	Origin	Limit	M_x	$\partial M/\partial H_c$	$\partial M/\partial R_c$	I
CD	C	0 to 3	$-H_c \times x$	$-x$	0	I
DB	D	0 to 3	$-H_c(3+x) - 60x$	$-(3+x)$	0	I
BA	B	0 to 4	$(-H_c \times 6) + (R_c \times x) - (60\times 3) - 40\frac{x^2}{2}$	-6	x	2I

$$(1) \Rightarrow \frac{\partial U}{\partial H_c} = \frac{1}{EI}\int_0^3 -H_c x(-x)\,dx + \frac{1}{EI}\int_0^3 (-H_c(3+x) - 60x) \cdot -(3+x)\,dx$$

$$+ \frac{1}{2I \times E}\int_0^4 (-6H_c + R_c x - 180 - 20x^2)(-6)\,dx$$

$$\frac{1}{EI}\left[\int_0^3 H_c x^2 \cdot dx + \int_0^3 (-H_c \times 3 - H_c \times x - 60x) \times -(3+x) \cdot dx \right.$$

$$\left. + \int_0^4 18H_c - 3R_c x + 540 + 60x^2 \cdot dx\right.$$

$$\frac{\partial U}{\partial H_C} = \frac{1}{EI}\left[\int_0^3 H_C x^2 \cdot dx + \int_0^3 (9H_C + 3H_C x + 180x + 3H_C x + H_C x^2 + 60x^2 \right.$$

$$\left. + \int_0^4 18H_C - 3R_C x + 540 + 60x^2 \cdot dx\right] = 0$$

$$= \frac{1}{EI}\left[\left[H_C \frac{x^3}{3}\right]_0^3 + \left[9H_C x + 3H_C \frac{x^2}{2} + \frac{180x^2}{2} + \frac{3H_C x^2}{2} + H_C \frac{x^3}{3} + \frac{60x^3}{3}\right]_0^3 + \left[18H_C x - \frac{3R_C x^2}{2} + 540x + \frac{60x^3}{3}\right]_0^4\right] = 0$$

$$= \frac{1}{EI}\left[\left(H_C \frac{3^3}{3}\right) + \left(9H_C \times 3 + 3H_C \times \frac{3^2}{2} + \frac{180 \times 3^2}{2} + 3H_C \times \frac{3^2}{2} + H_C \times \frac{3^3}{3} + \frac{60 \times 3^3}{3}\right) + \left(18H_C \times 4 - \frac{3R_C \cdot 4^2}{2} + (540 \times 4) + \frac{60 \times 4^3}{3}\right)\right] = 0$$

$$144H_C - 24R_C = -4790 \rightarrow (3)$$

$$\frac{\partial U}{\partial R_C} = 0, \Rightarrow \frac{1}{2EI}\int_0^4 (-H_C \times 6 + R_C x - 180 + 20x^2)(x)\,dx$$

$$\frac{1}{2EI}\int_0^4 (-H_C \times 6x + R_C x^2 - 180x + 20x^3)\,dx = 0$$

$$\frac{1}{2EI}\left[-H_C \times \frac{6x^2}{2} + R_C \frac{x^3}{3} - \frac{180x^2}{2} + \frac{20x^4}{4}\right]_0^4$$

$$\frac{1}{2EI}\left[-H_C \times \frac{6 \times 4^2}{2} + R_C \times \frac{4^3}{3} - \frac{180 \times 4^2}{2} + 20 \times \frac{4^4}{4}\right]$$

$-24H_C + 10.65 R_C = 1360 \rightarrow (4)$

By solving (3) & (4)

$H_C = -19.18\text{ kN } (\rightarrow)$

$R_C = 84.46\text{ kN}$

For CD:

$M_x = -H_C \times x \quad x = 0, M_C = 0.$

$M_D = -(-19.18 \times 3) = 57.54\text{ kNm}$

For DB:

$M_x = -H_C(3+x) - 60x$

$x = 0, \quad M_D = -(-19.18)(3+0) = 57.54\text{ kNm}$

$x = 3m, \quad M_B = -(-19.18)(3+3) - (60 \times 3)$

$= -64.92\text{ kNm}$

For BA:

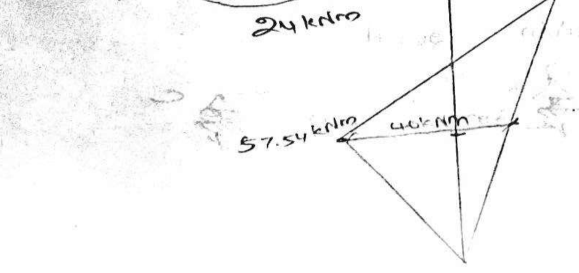
$M_x = (-H_C \times 6) + (R_C \times x) - (60 \times 3) - \frac{40x^2}{2}$

$x = 0, \quad M_B = -(-19.18) \times 6 + 84.46 \times 0 - 180 - \frac{40 \times 0}{2}$

$M_B = -64.92\text{ kNm}$

$x = 4, \quad M_A = -(-19.18 \times 6) + (84.46 \times 4) - 180 - \frac{40 \times 4^2}{2}$

$= -47.08\text{ kNm}$



$\frac{wl^2}{8} = \frac{10 \times 4^2}{8} = 24\text{ kNm}$

$\frac{Wl}{4} = \frac{60 \times 6}{4} = 90\text{ kNm}$

SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

ACADEMIC YEAR 2024-25

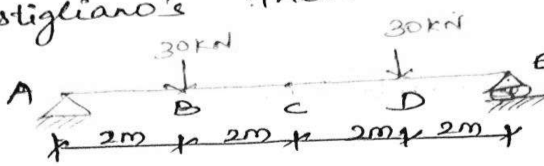
Department of Civil Engineering (SA I)

Assignment - 1

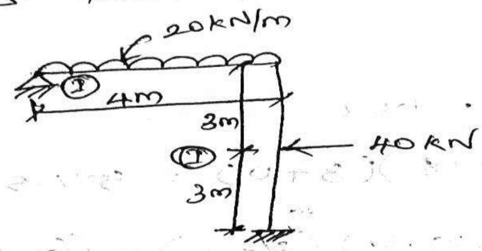
1. Calculate the vertical displacement of point using Castigliano's theorem.

$I = 600 \times 10^6$ mm⁴

$E = 200$ Gpa.

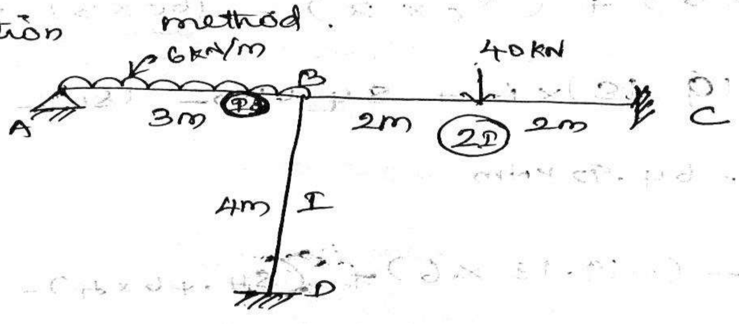


2. Analyse the frame as shown in figure by Castigliano's theorem.



Assignment - 2

1. Analyse the frame as shown in figure by slope deflection method.



2. Analyse the beam as shown in figure by slope deflection method. Support B settles by 30mm. Take $E = 2 \times 10^5$ N/mm², $I = 2 \times 10^9$ mm⁴.