# SNS COLLEGE OF TECHNOLOGY

**(An Autonomous Institution)**

**COIMBATORE-35**

**Accredited by NBA-AICTE and Accredited by NAAC – UGC with A++ Grade**
**Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai**

## DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING


**COURSE NAME: 19EEB301/ CONTROL SYSTEMS**

III YEAR / V SEMESTER

Unit I – SYSTEMS AND THEIR REPRESENTATIONS

# Topic : Mathematical modeling of Electrical and Mechanical systems

# Mathematical Models

- useful for analysis and design of control systems.
- Differential equation model is a time domain mathematical model of control systems.
- Transfer function model is an s-domain mathematical model of control systems.
- The Transfer function of a Linear Time Invariant (LTI) system is defined as the ratio of Laplace transform of output and Laplace transform of input by assuming all the initial conditions are zero.
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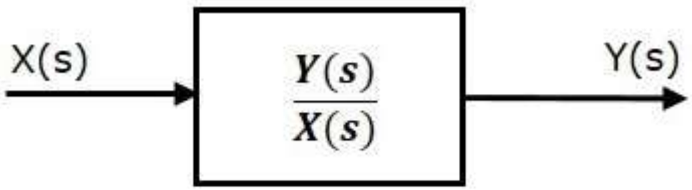



$$Transfer\ Function = \frac{Y(s)}{X(s)}$$

# Modeling of Translational Mechanical Systems

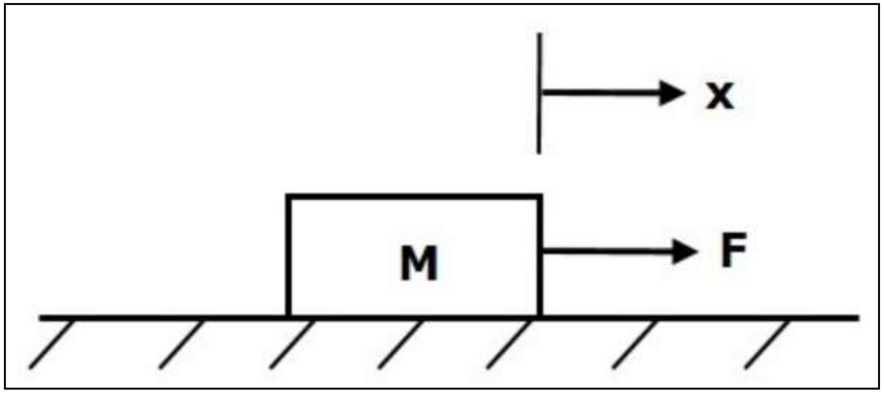
- Translational mechanical systems move along a straight line.
- Three basic elements
  - Mass
  - spring
  - dashpot or damper
- If a force is applied to a translational mechanical system, then it is opposed by opposing forces due to mass, elasticity and friction of the system.
- The applied force and the opposing forces are in opposite directions, the algebraic sum of the forces acting on the system is zero.

# Modeling of Translational Mechanical Systems

## Mass

- Mass is the property of a body, which stores kinetic energy.
- If a force is applied on a body having mass M, then it is opposed by an opposing force due to mass.
- This opposing force is proportional to the acceleration of the body.
- Assume elasticity and friction are negligible.

$$F_m \propto a$$




$$\Rightarrow F_m = Ma = M\frac{d^2x}{dt^2}$$

$$F = F_m = M\frac{d^2x}{dt^2}$$

Where,
**F** is the applied force
$\mathbf{F_m}$ is the opposing force due to mass
**M** is mass, **a** is acceleration
**x** is displacement

# Modeling of Translational Mechanical Systems

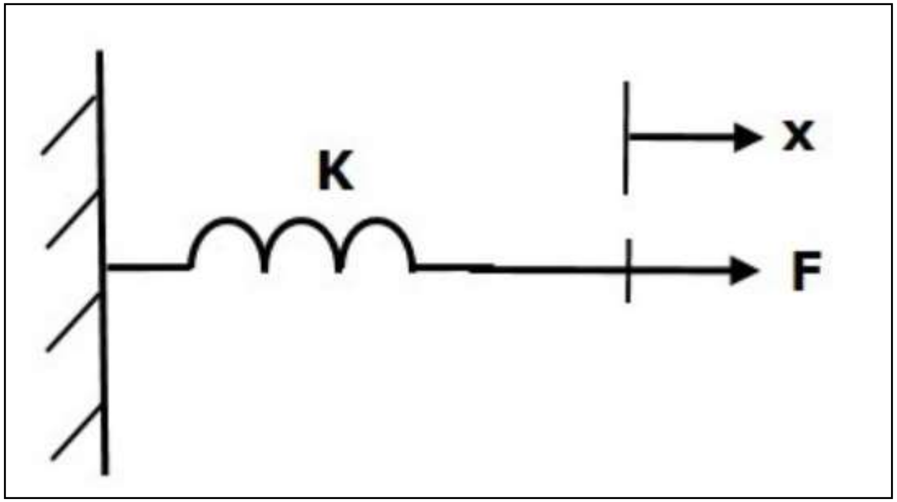
## Spring

- Spring is an element, which stores potential energy.
- If a force is applied on spring K, then it is opposed by an opposing force due to elasticity of spring.
- This opposing force is proportional to the displacement of the spring. Assume mass and friction are negligible.

$$F \propto x$$

$$\Rightarrow F_k = Kx$$

$$F = F_k = Kx$$



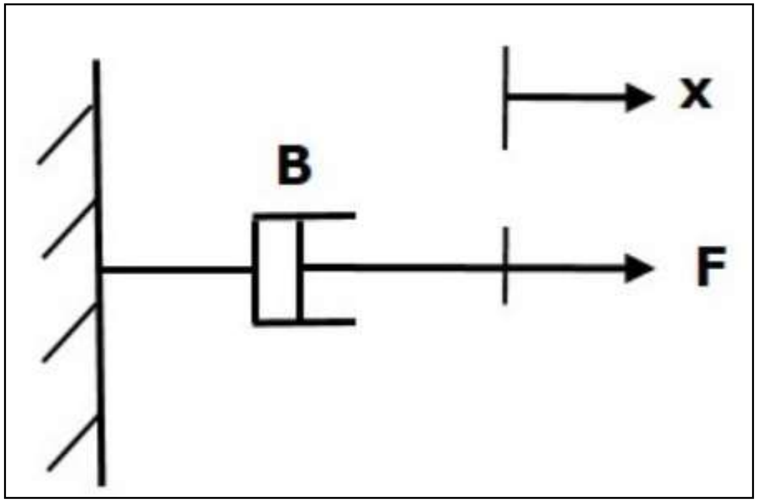

Where,
**F** is the applied force
$\mathbf{F_k}$ is the opposing force due to elasticity of spring
**K** is spring constant, **x** is displacement

# Modeling of Translational Mechanical Systems

## Dashpot

- If a force is applied on dashpot B, then it is opposed by an opposing force due to friction of the dashpot.
- This opposing force is proportional to the velocity of the body.
- Assume mass and elasticity are negligible.

$$F_b \propto \nu$$




$$\Rightarrow F_b = B\nu = B\frac{\mathrm{d}x}{\mathrm{d}t}$$

$$F = F_b = B\frac{\mathrm{d}x}{\mathrm{d}t}$$

Where,
**F** is the applied force
**$F_b$** is the opposing force due to friction of dashpot
**B** is the frictional coefficient
**v** is velocity, **x** is displacement

# Modeling of Rotational Mechanical Systems

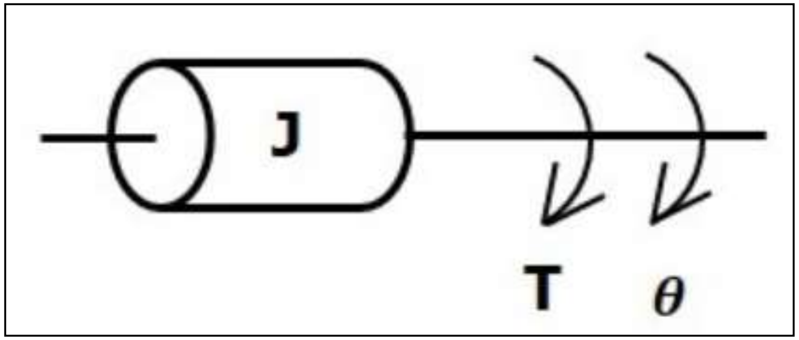
- Rotational mechanical systems move about a fixed axis.
- Three basic elements
  - Moment of inertia
  - Torsional spring
  - dashpot or damper
- If a torque is applied to a rotational mechanical system, then it is opposed by opposing torques due to moment of inertia, elasticity and friction of the system.
- The applied torque and the opposing torques are in opposite directions, the algebraic sum of torques acting on the system is zero.

# Modeling of Rotational Mechanical Systems

## Moment of Inertia

- moment of inertia stores kinetic energy.
- If a torque is applied on a body having moment of inertia J, then it is opposed by an opposing torque due to the moment of inertia.
- This opposing torque is proportional to angular acceleration of the body. Assume elasticity and friction are negligible.

$$T_j \propto \alpha$$




$$\Rightarrow T_j = J\alpha = J\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$$

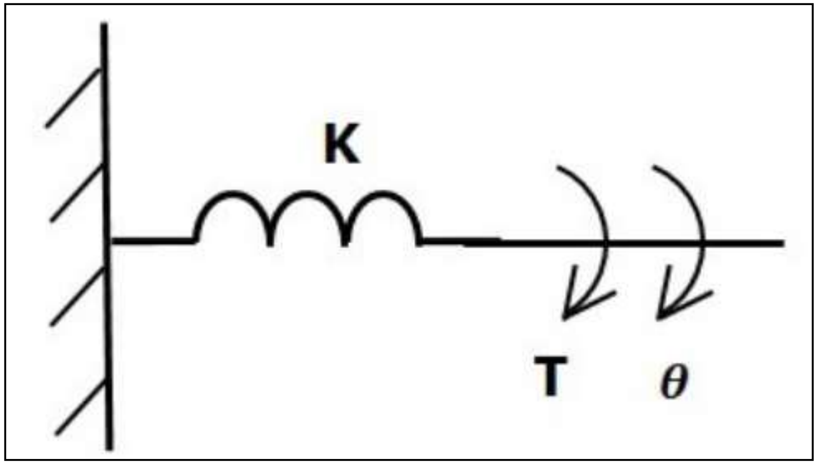
$$T = T_j = J\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$$

Where,
**T** is the applied torque
**$T_j$** is the opposing torque due to moment of inertia, **J** is moment of inertia
**$\alpha$** is angular acceleration
**$\theta$** is angular displacement

# Modeling of Rotational Mechanical Systems

## Torsional Spring

- torsional spring stores potential energy.
- If a torque is applied on torsional spring K, then it is opposed by an opposing torque due to the elasticity of torsional spring.
- This opposing torque is proportional to the angular displacement of the torsional spring. Assume that the moment of inertia and friction are negligible.




$$T_k \propto \theta$$

$$\Rightarrow T_k = K\theta$$

$$T = T_k = K\theta$$

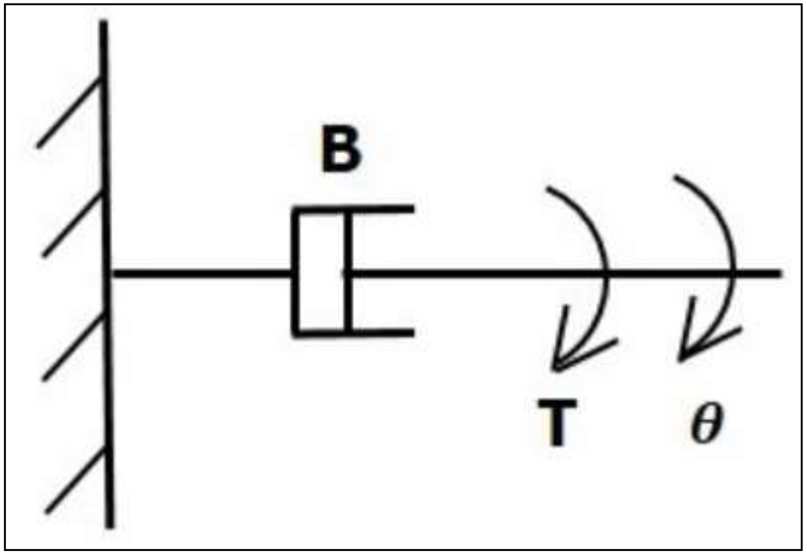
Where,
**T** is the applied torque
$\mathbf{T_k}$ is the opposing torque due to elasticity of torsional spring
**K** is the torsional spring constant
**θ** is angular displacement

# Modeling of Rotational Mechanical Systems

## Dashpot

- If a torque is applied on dashpot B, then it is opposed by an opposing torque due to the rotational friction of the dashpot.
- This opposing torque is proportional to the angular velocity of the body. Assume the moment of inertia and elasticity are negligible.

$$T_b \propto \omega$$




$$\Rightarrow T_b = B\omega = B\frac{d\theta}{dt}$$

$$T = T_b = B\frac{d\theta}{dt}$$

Where,

**T** is the applied torque

**$T_b$** is the opposing torque due to the rotational friction of the dashpot

**B** is the rotational friction coefficient

**ω** is the angular velocity

**θ** is angular displacement

Thank You