THEORY OF INFERENCE FOR PREDICATE CALCULUS:

i) Universal Specification (US rule): $\forall x, P(x) \Rightarrow P(y)$

ii) Universal Generalization (UG rule): $P(y) \Rightarrow \forall x, P(x)$

iii) Existential Specification (ES rule): $\exists x, P(x) \Rightarrow P(y)$

iv) Existential Generalization (EG rule): $P(y) \Rightarrow \exists x, P(x)$

(1) Show that the premises, "one student in this class knows how to write programs in JAVA" and "Everyone who knows how to write program in JAVA can get a high-paying Job". Imply the conclusion, "someone in this class can get a high-paying job".

Soln:

Let $A(x)$: x is in this class.

$J(x)$: x knows how to write program in JAVA

$H(x)$: x can get a high paying job.

Given premises are:

$$\exists(x)\,(A(x) \wedge J(x)) \; ; \; \forall(x)\,(J(x) \rightarrow H(x))$$

Conclusion: $\exists(x)\,(A(x) \wedge H(x))$

Step 1:	$\exists(x)\,(A(x) \wedge J(x))$	Rule P
Step 2:	$A(y) \wedge J(y)$	Rule ES
Step 3:	$A(y)$	Rule T_2 [simplification rule]
Step 4:	$J(y)$	Rule T_2 [" $A(y) \wedge J(y) \Rightarrow A(y), J(y)$]
Step 5:	$\forall(x)\,(J(x) \to H(x))$	Rule P
Step 6:	$J(y) \to H(y)$	Rule US
Step 7:	$H(y)$	Rule $T_{4,6}$ [Modus phones $J(y), J(y) \to H(y) \Rightarrow H(y)$]
Step 8:	$A(y) \wedge H(y)$	Rule $T_{3,7}$ [simplification rule $A(y), H(y) \Rightarrow A(y) \wedge H(y)$
Step 9:	$\exists(x)\,(A(x) \wedge H(x))$	Rule EG

② Verify the validity of the following argument.
"Every living thing is a plant or an animal". "Johns gold fish is alive and it is not a plant". "All animals have hearts". Therefore, "John's gold fish has a heart".

Soln:

$L(x)$: x is a living thing. $L(j)$: j is alive
$P(x)$: x is a plant $P(j)$: j is not a plant
$A(x)$: x is a animal
$H(x)$: x is a heart. $H(j)$: j has a heart.

Given premises are: $\forall(x)\ (L(x) \to P(x) \vee A(x))$;
$L(j) \wedge \neg P(j)$;
$\forall(x)\ (A(x) \to H(x))$

Conclusion: $H(j)$.

Step 1: $\forall(x)\ (L(x) \to P(x) \vee A(x))$ — Rule P
Step 2: $L(j) \to P(j) \vee A(j)$ — Rule US
Step 3: $L(j) \wedge \neg P(j)$ — Rule P
Step 4: $L(j)$ — Rule T_3 [simplification $L(j) \wedge \neg P(j) \Rightarrow L(j)$]
Step 5: $P(j) \vee A(j)$ — Rule $T_{2,4}$ [Modus phones $L(j), L(j) \to P(j) \vee A(j) \Rightarrow P(j) \vee A(j)$]
Step 6: $\neg P(j) \to A(j)$ — Rule T_5 [Material Implic]
Step 7: $\forall(x)\ (A(x) \to H(x))$ — Rule P
Step 8: $A(j) \to H(j)$ — Rule US
Step 9: $\neg P(j) \to H(j)$ — Rule $T_{6,8}$ [chain rule]
Step 10: $\neg P(j)$ — Rule T_3 [simplification rule.
Step 11: $H(j)$ — Rule $T_{9,10}$ [Modus phon

(3) "All rock music is loud music", "Some rock music exist" Therefore "Some loud music exist".

$R(x)$: x is a rock music

$L(x)$: x is a loud music

Given premises are:

$$\forall(x)\,(R(x) \rightarrow L(x))\;;\;\exists(x)\,R(x)$$

Conclusion: $\exists(x)\,L(x)$

Step 1:	$\forall(x)(R(x) \rightarrow L(x))$	Rule P
Step 2:	$R(y) \rightarrow L(y)$	Rule US
Step 3:	$\exists(x)\,R(x)$	Rule P
Step 4:	$R(y)$	Rule ES
Step 5:	$L(y)$	Rule $T_{2,4}$ [Modus phones] $R(y), R(y) \rightarrow L(y) \Rightarrow L(y)$
Step 6:	$\exists(x)\,L(x)$	Rule EG.