SNS COLLEGE OF TECHNOLOGY

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COURSE CODE& NAME : *19CSB301* &AUTOMATA THEORY AND COMPILER DESIGN

III YEAR/ V SEMESTER

UNIT – I FINITE AUTOMATA AND REGULAR LANGUAGES

Topic: Pushdown Automata

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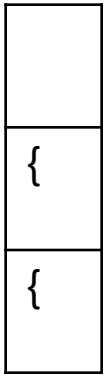
PDA – Push Down Automata

A Push Down Automata(PDA) is a way to implement a context free Grammar in a similar way to design Finite Automata for Regular Grammar

Grammar Type	Language Accepted	Automaton
Type 0	Recursively enumerable language	Turing Machine
Type 1	Context-sensitive language	Linear-bounded automaton
Type 2	Context-free language	Pushdown automaton
Type 3	Regular language	Finite state automaton

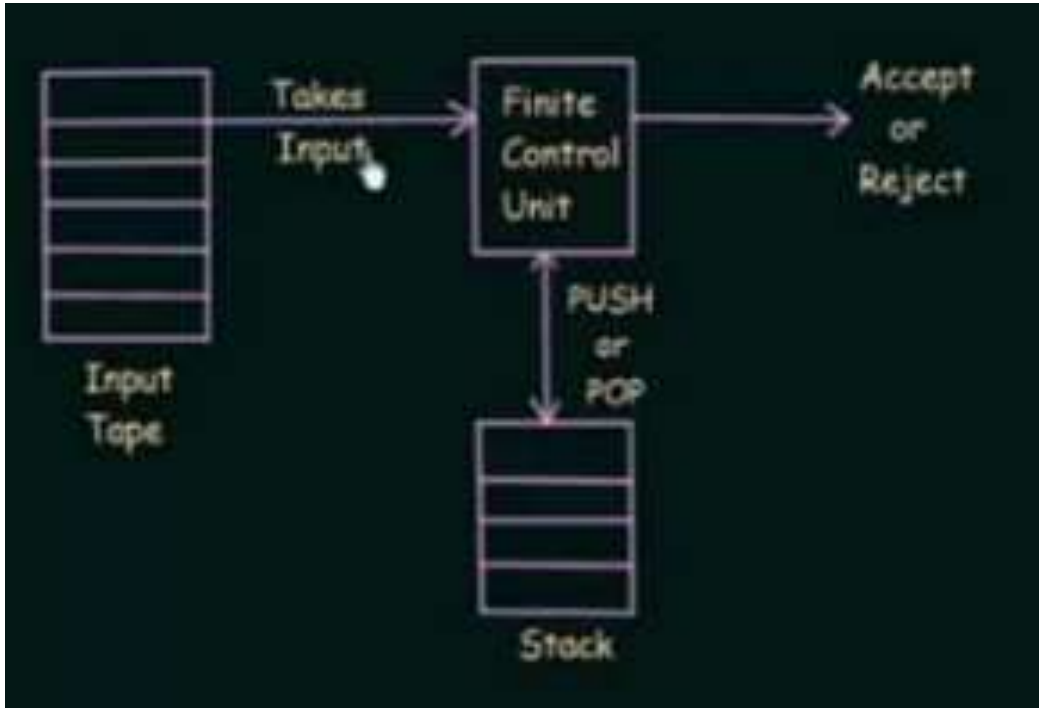
- FSA
  - not applicable for all domains
  - Limited Memory
- PDA
  - It is more Powerful than Finite State Machine
  - PDA has more memory
  - **FSA + Stack**
  - Applications
    - Calculator
    - Java / C Program

Stack



Components of PDA

- Input Tape
- Finite Control Unit
- Stack



Formal Definition of PDA

Pushdown Automata (Formal Definition)

A Pushdown Automata is formally defined by 7 Tuples as shown below:

$P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

where,

- Q = A finite set of States
- Σ = A finite set of Input Symbols
- Γ = A finite Stack Alphabet
- δ = The Transition Function
- q_0 = The Start State
- z_0 = The Start Stack Symbol
- F = The set of Final / Accepting States

δ takes as argument a triple $\delta (q, a, X)$ where:

(i) q is a State in Q

(ii) a is either an Input Symbol in Σ or $a = \epsilon$

(iii) X is a Stack Symbol, that is a member of Γ

Formal Definition of PDA

δ = The Transition Function

q_0 = The Start State

z_0 = The Start Stack Symbol

F = The set of Final / Accepting States

δ takes as argument a triple δ (q, a, X) where:

(i) q is a State in Q

(ii) a is either an Input Symbol in Σ or a = ∈

(iii) X is a Stack Symbol, that is a member of Γ

The output of δ is finite set of pairs (p, γ) where:

p is a new state

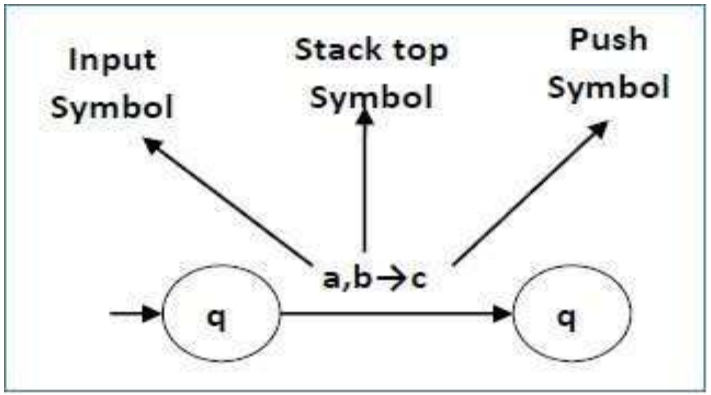
γ is a string of stack symbols that replaces X at the top of the stack

Eg. If γ = ∈ then the stack is popped

If γ = X then the stack is unchanged

If γ = YZ then X is replaced by Z and Y is pushed onto the stack

- The following diagram shows a transition in a PDA from a state q_1 to state q_2, labeled as a,b → c –



- This means at state **q_1**, if we encounter an input string **'a'** and top symbol of the stack is **'b'**, then we pop **'b'**, push **'c'** on top of the stack and move to state **q_2**.

Terminologies Related to PDA

- The "turnstile" notation is used for connecting pairs of ID's that represent one or many moves of a PDA. The process of transition is denoted by the turnstile symbol "⊢".
- Consider a PDA (Q, ∑, S, δ, q_0, I, F). A transition can be mathematically represented by the following turnstile notation –

```
(p, aw, Tβ) ⊢ (q, w, αb)
```

- (p, aw, Tβ) ⊢ (q, w, αb) This implies that while taking a transition from state **p** to state **q**, the input symbol **'a'** is consumed, and the top of the stack **'T'** is replaced by a new string **'α'**.
- **Note** – If we want zero or more moves of a PDA, we have to use the symbol (⊢*) for it.

Language of PDA- Final State Acceptability

In final state acceptability, a PDA accepts a string when, after reading the entire string, the PDA is in a final state. From the starting state, we can make moves that end up in a final state with any stack values. The stack values are irrelevant as long as we end up in a final state.

For a PDA $(Q, \sum, S, \delta, q_0, I, F)$, the language accepted by the set of final states F is −

$L(PDA) = \{w \mid (q_0, w, I) \vdash^* (q, \varepsilon, x), q \in F\}$

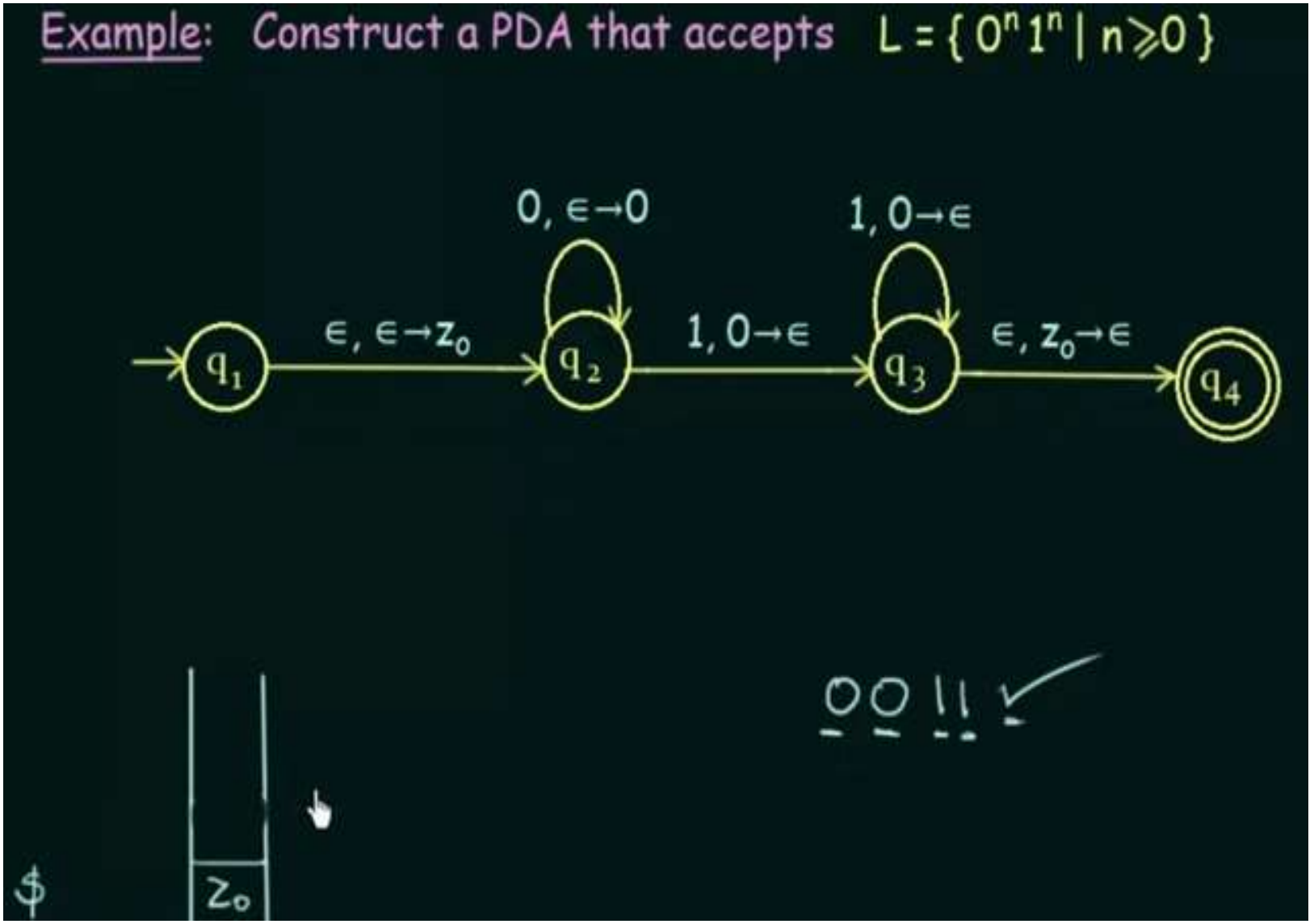
for any input stack string **x**.

Language of PDA- Empty Stack Acceptability

Here a PDA accepts a string when, after reading the entire string, the PDA has emptied its stack.

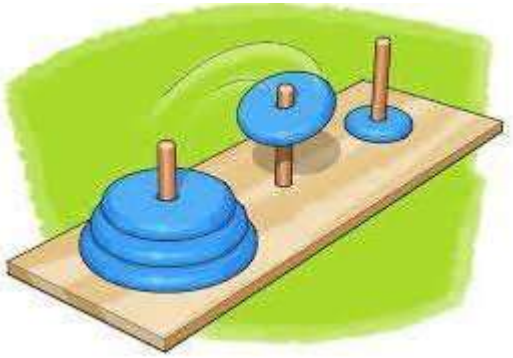
For a PDA $(Q, \sum, S, \delta, q_0, I, F)$, the language accepted by the empty stack is −

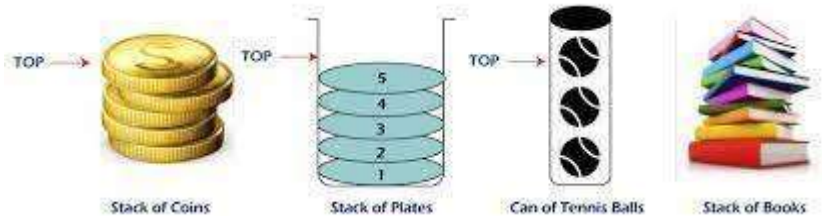
$L(PDA) = \{w \mid (q_0, w, I) \vdash^* (q, \varepsilon, \varepsilon), q \in Q\}$


Example: Construct a PDA that accepts $L = \{ 0^n 1^n \mid n \geq 0 \}$
$0, \epsilon \rightarrow 0$
$1, 0 \rightarrow \epsilon$
$\epsilon, \epsilon \rightarrow Z_0$
$1, 0 \rightarrow \epsilon$
$\epsilon, Z_0 \rightarrow \epsilon$
q_1
q_2
q_3
q_4
00 !!
$
Z_0

PDA Applications

- Syntax Analysis phase in Compiler
- Towers of Hanoi
- Smart phone – calculator
- Stack Applications





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