SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

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DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

UNIT II FINITE IMPULSE RESPONSE FILTERS

- Introduction to FIR
- Linear phase FIR filter
- FIR filter design using window method
- Low Pass Filter
- Frequency sampling method
- Realization of FIR filter using direct form 1, Direct form 2
- Realization of FIR filter using Cascade structures
- Realization of FIR filter using parallel structures

Example 6.8 Design an ideal bandreject filter with a desired frequency response

$$H_d(e^{j\omega}) = 1 \quad \text{for} \quad |\omega| \leq \frac{\pi}{3} \quad \text{and} \quad |\omega| \geq \frac{2\pi}{3}$$
$$= 0 \quad \text{otherwise}$$

Find the value of $h(n)$ for $N = 11$. Find $H(z)$. Plot the magnitude response.

Solution

The desired frequency response is shown in Fig. 6.14.

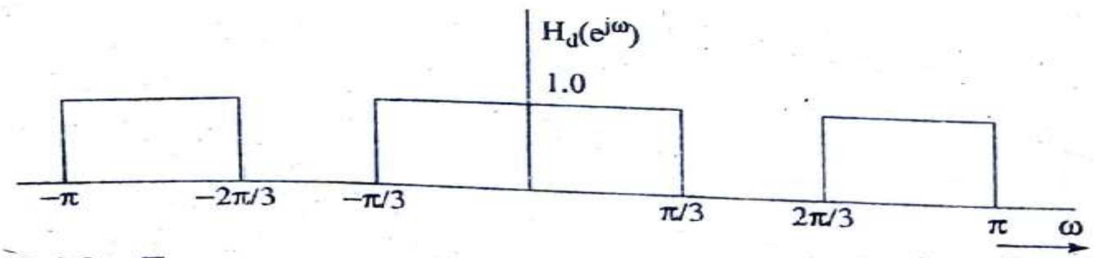

Fig. 6.14 Frequency response of Bandreject filter of example 6.8.

We know

$$
\begin{aligned}
h_d(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega \\
&= \frac{1}{2\pi}\left[\int_{-\pi}^{-2\pi/3} e^{j\omega n}d\omega + \int_{-\pi/3}^{\pi/3} e^{j\omega n}d\omega + \int_{2\pi/3}^{\pi} e^{j\omega n}d\omega\right] \\
&= \frac{1}{2\pi jn}\left[e^{-j2\pi n/3} - e^{-j\pi n} + e^{j\pi n/3} - e^{-j\pi n/3} + e^{j\pi n} - e^{j2\pi n/3}\right] \\
&= \frac{1}{\pi n}\left[\sin \pi n + \sin\frac{\pi}{3}n - \sin\frac{2\pi}{3}n\right] \qquad -\infty \le n \le \infty
\end{aligned}
$$

Truncating $h_d(n)$ to 11 samples, we have

$$
\begin{aligned}
h(n) &= h_d(n) \quad \text{for} \quad |n| \le 5 \\
&= 0 \quad \text{otherwise}
\end{aligned}
$$

The filter coefficients are symmetrical about $n = 0$ satisfying the condition $h(n) = h(-n)$.

For $n = 0$

$$h(0) = \lim_{n \to 0} \left[\frac{\sin \pi n}{\pi n} + \frac{\sin \frac{\pi}{3} n}{\pi n} - \frac{\sin \frac{2\pi}{3} n}{\pi n} \right]$$

$$= \left(1 + \frac{1}{3} - \frac{2}{3}\right) = 0.667$$

$$h(1) = h(-1) = \frac{\sin \pi + \sin \frac{\pi}{3} - \sin \frac{2\pi}{3}}{\pi} = 0$$

$$h(2) = h(-2) = \frac{\sin 2\pi + \sin \frac{2\pi}{3} - \sin \frac{4\pi}{3}}{2\pi.} = 0.2757$$

$$h(3) = h(-3) = \frac{\sin 3\pi + \sin \pi - \sin 2\pi}{3\pi} = 0$$

$$h(4) = h(-4) = \frac{\sin 4\pi + \sin \frac{4\pi}{3} - \sin \frac{8\pi}{3}}{4\pi} = -0.1378$$

$$h(5) = h(-5) = \frac{\sin 5\pi + \sin \frac{5\pi}{3} - \sin \frac{10\pi}{3}}{5\pi} = 0$$

The transfer function of the filter is

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} [h(n)(z^n + z^{-n})]$$

$$= 0.667 + 0.2757(z^2 + z^{-2}) - 0.1378(z^4 + z^{-4}) \qquad (6.67)$$

The transfer function of the realizable filter

$$H'(z) = z^{-5}H(z)$$
$$= -0.1378z^{-1} + 0.2757z^{-3} + 0.667z^{-5} + 0.2757z^{-7} - 0.1378z^{-9}$$

The filter coefficients of the causal filter are

$$h(0) = h(10) = h(2) = h(8) = h(4) = h(6) = 0$$
$$h(1) = h(9) = -0.1378$$
$$h(3) = h(7) = 0.2757$$
$$h(5) = 0.667$$
$$\overline{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n)\cos\omega n$$
$$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.667$$
$$a(n) = 2h\left(\frac{N-1}{2} - n\right)$$
$$a(1) = 2h(5-1) = 2h(4) = 0$$
$$a(2) = 2h(5-2) = 2h(3) = 0.5514$$
$$a(3) = 2h(5-3) = 2h(2) = 0$$
$$a(4) = 2h(5-4) = 2h(1) = -0.2756$$
$$a(5) = 2h(5-5) = 2h(0) = 0$$

ω (in degrees)	0	15	30	45	60	75
$\overline{H}(e^{j\omega})$	0.9428	1.0067	1.08	0.9426	0.529	0.0516
$\|H(e^{j\omega})\|_{dB}$	−0.5	0.058	0.67	−0.513	−5.53	−25.7

90	105	120	135	150	165	180
16	0.0516	0.529	0.9426	1.08	1.0067	0.9428
−15.9	−25.7	−5.53	−0.513	0.67	0.058	−0.5