



# **SNS COLLEGE OF TECHNOLOGY**

**Coimbatore-35**

**An Autonomous Institution**



Accredited by NBA – AICTE and Accredited by NAAC – UGC with ‘A++’(III Cycle) Grade  
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

## **DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING**

### **23ECB201- DIGITAL SYSTEM DESIGN**

**II YEAR/ III SEMESTER**

**UNIT 1 – BOOLEAN THEOREMS AND LOGIC REDUCTION**

**TOPIC – NUMBER SYSTEMS**



# NUMBER SYSTEM



- A number system is simply a way of representing numeric values.
- It uses symbols called numerals to represent numeric quantities.
- A digital system can understand positional number system only where there are a few symbols called digits and these symbols represent different values depending on the position they occupy in the number.
- A value of each digit in a number can be determined using
  - The digit
  - The position of the digit in the number
  - The base of the number system (where base is defined as the total number of digits available in the number system).



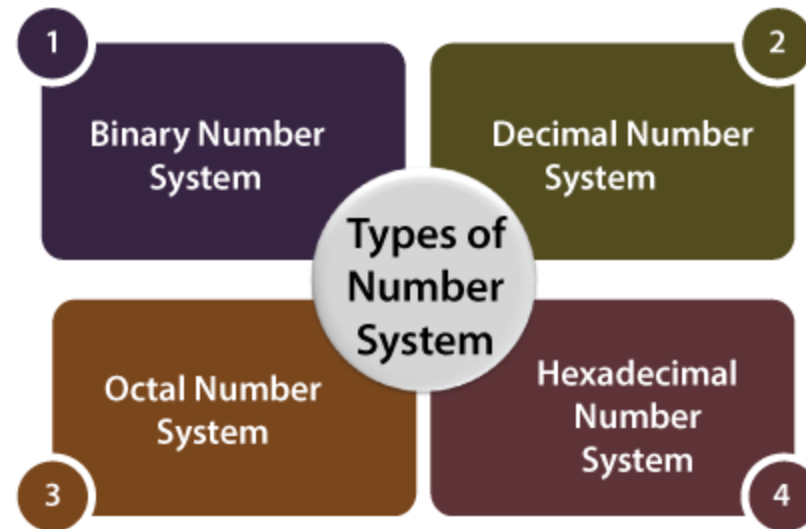
In the digital computer, there are various types of number systems used for representing information.

Binary Number System

Decimal Number System

Hexadecimal Number System

Octal Number System





# Decimal Number System



- The number system that we use in our day-to-day life is the decimal number system.
- Decimal number system has base 10 as it uses 10 digits from 0 to 9.
- In decimal number system, the successive positions to the left of the decimal point represents units, tens, hundreds, thousands and so on.
- Each position represents a specific power of the base (10).
- For example, the decimal number 1234 is written as  $(1 \times 1000) + (2 \times 100) + (3 \times 10) + (4 \times 1) = (1 \times 10^3) + (2 \times 10^2) + (3 \times 10^1) + (4 \times 1) = 1000 + 200 + 30 + 1$  1234



# Binary Number System

- A binary number system is used in the digital computers.
- Each digit is known as a bit.
- Uses only two digits, either 0 or 1.
- The location of a digit in a binary number represents a specific power of the base (2) of the number system.
- Two types of electronic pulses present in a binary number system are the absence of an electronic pulse representing '0' and second one is the presence of electronic pulse representing '1'.
- A four-bit collection (1101) is known as a nibble, and a collection of eight bits (11001010) is known as a byte.



# Octal Number System

- The octal number system has base 8 (means it has only eight digits from 0 to 7).
- Also called base 8 number system
- The location of a digit in an octal number represents a specific power of the base (8) of the number system.
- Octal base  $8=2^3$  every 3-bit group of binary can be represented by an octal digit.
- Each set of bits has a distinct value between 0 and 7.
- An octal number is thus, 1/3rd the length of the corresponding binary number.



# Hexadecimal Number System



- Uses 10 digits and 6 letters, 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F.
- Letters represents numbers starting from 10. A = 10, B = 11, C = 12, D = 13, E = 14, F = 15.
- Also called base 16 number system.
- This is a alphanumeric number system because it uses both alphabets and numerical to represent a hexadecimal number.
- The location of a digit in a hexadecimal number represents a specific power of the base (16) of the number system.
- **Examples:**

$(FAC2)_{16}$ ,  $(564)_{16}$ ,  $(0ABD5)_{16}$ ,  $(1123)_{16}$ ,  $(11F3)_{16}$ .



- Common Number System:

System	Base	Symbols
Decimal	10	0, 1, ... 9
Binary	2	0, 1
Octal	8	0,1,2,,...7
Hexadecimal	16	0, 1, ... 9, A, B, ... F





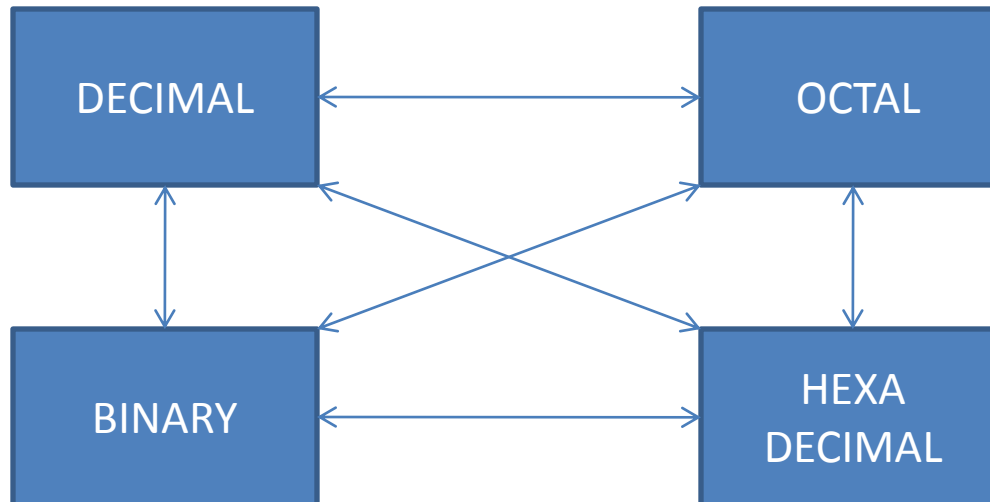
# COUNTING METHODS



Decimal	Binary	Octal	Hexadecimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F



# Conversion among the Number system





# Binary to Octal Conversion



- Group into 3's starting at least significant symbol (if the number of bits is not evenly divisible by 3, then add 0's at the most significant end) •
- write 1 octal digit for each group.

e.g.:  $(1010101)_2$  to  $( )_8$

001 010 101



1 2 5

Answer =  $(125)_8$



**Example :  $(111110101011.0011)_2$**

1. Firstly, we make pairs of three bits on both sides of the binary point.

111110101011.0011

111/110/101/011.001/1

On the right side of the binary point, the last pair has only one bit. To make it a complete pair of three bits, we added two zeros on the extreme side.

111    110    101    011. 001    100

2. Then, we wrote the octal digits, which correspond to each pair.

**$(111110101011.0011)_2 = (7653.14)_8$**



# Octal to Binary Conversion



- For each of the Octal digit write its binary equivalent

e.g.:  $(257)_8$  to  $( )_2$

2  $\longrightarrow$  010

5  $\longrightarrow$  101

7  $\longrightarrow$  111

Answer= $(10101111)_2$

**Example :  $(152.25)_8$**

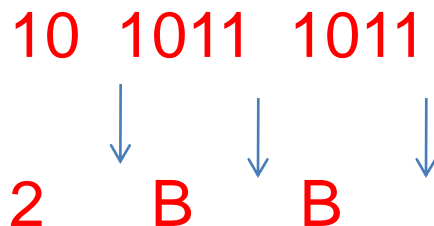
- We write the three-bit binary digit for 1, 5, 2, and 5.
- $(152.25)_8 = (001101010.010101)_2$
- The binary number of the octal number 152.25 is  $(001101010.010101)_2$



# Binary to Hexadecimal Conversion

- In a binary number, the pair of four bits is equal to one hexadecimal digit
- Group into 4's starting at least significant symbol (if the number of bits is not evenly divisible by 4, then add 0's at the most significant end)
- write 1 hex digit for each group.

e.g.:  $(1010111011)_2$  to  $( )_{16}$



Answer=  $(2BB)_{16}$



**Example :  $(10110101011.0011)_2$**

1. Firstly, we make pairs of four bits on both sides of the binary point.

11110101011.0011

111 1010 1011.0011

- On the left side of the binary point, the first pair has three bits. To make it a complete pair of four bits, add one zero on the extreme side.

0111 1010 1011.0011

2. Then, we write the hexadecimal digits, which correspond to each pair.

**$(011110101011.0011)_2 = (7AB.3)_{16}$**



# Hexadecimal to Binary Conversion

- For each of the Hex digit write its binary equivalent (use 4 bits to represent).

e.g.:  $(25A0)_{16}$  to  $( )_2$

2  $\longrightarrow$  0010

5  $\longrightarrow$  0101

A  $\longrightarrow$  1010

0  $\longrightarrow$  0000

Answer=  $(10\ 0101\ 1010\ 0000)_2$





### Example : $(152A.25)_{16}$

- We write the four-bit binary digit for 1, 5, A, 2, and 5.
- $(152A.25)_{16} = (0001\ 0101\ 0010\ 1010.0010\ 0101)_2$
- The binary number of the hexadecimal number 152.25 is  $(1010100101010.00100101)_2$



# Octal to Hexadecimal Conversion

Steps:

- 1. Convert octal number to its binary equivalent
- 2. Convert binary number to its hexadecimal equivalent

e.g.:  $(635.27)_8$  to  $( )_{16}$

6      3      5      .      2      7

0001    1001    1101    0101    1100    .    ↓    STEP 1

1      9      D      .      5      C      .    ↓    STEP 2



**Example :  $(152.25)_8$**

**Step 1:**

- We write the three-bit binary digit for 1, 5, 2, and 5 and 8

$$(152.25)_8 = (001101010.010101)_2$$

- The binary number of the octal number 152.25 is  **$(001101010.010101)_2$**

**Step 2:**

1. Now, we make pairs of four bits on both sides of the binary point.

$$0 \quad 0110 \quad 1010.0101 \quad 01$$

- On the left side of the binary point, the first pair has only one digit, and on the right side, the last pair has only two-digit. To make them complete pairs of four bits, add zeros on extreme sides.

$$0000 \quad 0110 \quad 1010.0101 \quad 0100$$

2. Now, we write the hexadecimal digits, which correspond to each pair.

$$(0000 \quad 0110 \quad 1010.0101 \quad 0100)_2 = (6A.54)_{16}$$

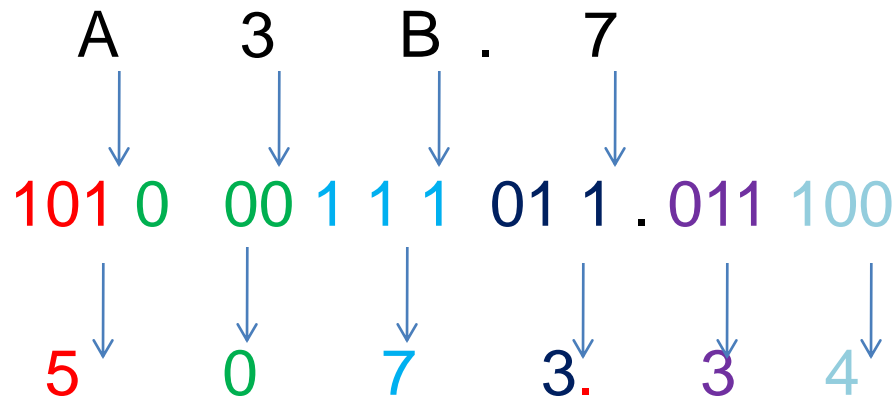


# Hexadecimal to Octal Conversion



- Steps:
  - 1. Convert hexadecimal number to its binary equivalent
  - 2. Convert binary number to its octal equivalent

e.g.:  $(A3B.7)_{16}$  to  $( )_8$



STEP-1

STEP-2

**Answer=  $(5073.34)_8$**



**Example :  $(152A.25)_{16}$**

**Step 1:**

- We write the four-bit binary digit for 1, 5, 2, A, and 5.

$$(152A.25)_{16} = (0001\ 0101\ 0010\ 1010.0010\ 0101)_2$$

- The binary number of hexadecimal number 152A.25 is  $(0011010101010.010101)_2$

**Step 2:**

- Then, we make pairs of three bits on both sides of the binary point.

$$001\ 010\ 100\ 101\ 010.001\ 001\ 010$$

- Then, we write the octal digit, which corresponds to each pair.

$$(001010100101010.001001010)_2 = (12452.112)_8$$

- The octal number of the hexadecimal number 152A.25 is **12452.112**



# Any Base to Decimal Conversion

- Converting from any base to decimal is done by multiplying each digit by its weight and summing.

Example:

- Binary to Decimal Conversion:  $(1011.11)_2$

$$\begin{aligned}(1011.11)_2 &= (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1}) \\ &\quad + (1 \times 2^{-2}) \\ &= 8 + 0 + 2 + 1 + 0.5 + 0.25 \\ &= 11.75_{10}\end{aligned}$$



# Octal to Decimal Conversion

- The process of converting octal to decimal is the same as binary to decimal. The process starts from multiplying the digits of octal numbers with its corresponding positional weights. And lastly, we add all those products.

**Example:  $(152.25)_8$**

**Step 1:**

- We multiply each digit of **152.25** with its respective positional weight, and last we add the products of all the bits with its weight.

- $(152.25)_8 = (1 \times 8^2) + (5 \times 8^1) + (2 \times 8^0) + (2 \times 8^{-1}) + (5 \times 8^{-2})$

$$(152.25)_8 = 64 + 40 + 2 + (2 \times 1/8) + (5 \times 1/64)$$

$$(152.25)_8 = 64 + 40 + 2 + 0.25 + 0.078125$$

$$(152.25)_8 = 106.328125$$

- The decimal number of the octal number 152.25 is **106.328125**



# Hexa-decimal to Decimal Conversion



- The process of converting hexadecimal to decimal is the same as binary to decimal. The process starts from multiplying the digits of hexadecimal numbers with its corresponding positional weights. And lastly, we add all those products.

**Example :  $(152A.25)_{16}$**

## **Step 1:**

- We multiply each digit of **152A.25** with its respective positional weight, and last we add the products of all the bits with its weight.

$$(152A.25)_{16} = (1 \times 16^3) + (5 \times 16^2) + (2 \times 16^1) + (A \times 16^0) + (2 \times 16^{-1}) + (5 \times 16^{-2})$$

$$(152A.25)_{16} = (1 \times 4096) + (5 \times 256) + (2 \times 16) + (10 \times 1) + (2 \times 16^{-1}) + (5 \times 16^{-2})$$

$$(152A.25)_{16} = 4096 + 1280 + 32 + 10 + (2 \times 1/16) + (5 \times 1/256)$$

$$(152A.25)_{16} = 5418 + 0.125 + 0.125$$

$$(152A.25)_{16} = 5418.14453125$$

- The decimal number of the hexadecimal number 152A.25 is **5418.14453125**





# Decimal to Any Base Conversion



- **Steps:**
  - **Convert integer part ( Successive Division Method )**
  - **Convert fractional part ( Successive Multiplication Method )**
- **Steps in Successive Division Method**
  - Divide the integer part of decimal number by desired base number, store quotient (Q) and remainder (R)
  - Consider quotient as a new decimal number and repeat step1 until quotient becomes 0
  - List the remainders in the reverse order
- **Steps in Successive Multiplication Method**
  - Multiply the fractional part of decimal number by desired base number
  - Record the integer part of product as carry and fractional part as new fractional part
  - Repeat steps 1 and 2 until fractional part of product becomes 0 or until you have many digits as necessary for your application
  - Read carries downwards to get desired base number



# Decimal to Binary Conversion



- Convert  $(25.035)_{10}$  to binary number
- **Step 1 Converting integral part : 25**

2		25	
2		12	1 ← First remainder
2		6	0 ← Second Remainder
2		3	0 ← Third Remainder
2		1	1 ← Fourth Remainder
		0	1 ← Fifth Reaminder

Read Up

Binary Number = 11001

- **Step 2 Converting fractional part: 0.35**

$0.35 \times 2 = 0.70$	with a carry of	0	Binary Point
$0.70 \times 2 = 1.40$	with a carry of	1	
$0.40 \times 2 = 0.80$	with a carry of	0	Read Down
$0.80 \times 2 = 1.60$	with a carry of	1	
$0.60 \times 2 = 1.20$	with a carry of	1	

- **Answer =  $(11001.01011)_2$**



# Decimal to Octal Conversion



- First step - Perform the division operation on the integer and the successive quotient with the base of octal(8).
- Second step - Perform the multiplication on the integer and the successive quotient with the base of octal(8).

**Example :  $(152.25)_{10}$**

## Step 1:

- Divide the number 152 and its successive quotients with base 8.

Operation	Quotient	Remainder
152/8	19	0
19/8	2	3
2/8	0	2

**$(152)_{10}=(230)_8$**



## Step 2:

- Now perform the multiplication of 0.25 and successive fraction with base 8.

Operation	Result	carry
$0.25 \times 8$	0	2

- $(0.25)_{10} = (2)_8$
- So, the octal number of the decimal number 152.25 is **230.2**



# Decimal to Hexadecimal conversion



First step - Perform the division operation on the integer and the successive quotient with the base of hexadecimal (16).

- Second step - Perform the multiplication on the integer and the successive quotient with the base of hexadecimal (16).

**Example :  $(152.25)_{10}$**

**Step 1:**

- Divide the number 152 and its successive quotients with base 16.

Operation	Quotient	Remainder
152/16	9	8
9/16	0	9

$$(152)_{10} = (98)_{16}$$



## Step 2:

- Now perform the multiplication of 0.25 and successive fraction with base 16.

Operation	Result	carry
$0.25 \times 16$	0	4

- $(0.25)_{10} = (4)_{16}$
- The hexadecimal number of the decimal number 152.25 is  $(230.4)_{16}$



## Practice Problems:

1. Convert Decimal to Binary  $(10.25)_{10}$
2. Convert Binary to Decimal  $(1010.01)_2$
3. Find the octal equivalent of the given hexadecimal number  $(1E.53)_{16}$
4. Find the hexadecimal equivalent of the given octal number  $(651.124)_8$



**THANK YOU**