SNS COLLEGE OF TECHNOLOGY

Coimbatore-35

An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' (III Cycle)Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

23ECB201- DIGITAL SYSTEMS DESIGN

II YEAR/ III SEMESTER

UNIT 1 – BOOLEAN THEOREMS AND LOGIC REDUCTION

TOPIC – BASIC THEOREMS AND PROPERTIES OF BOOLEAN ALGEBRA

BOOLEAN ALGEBRA

- In 1854 George Boole introduced a systematic treatment of logic and developed an algebraic system known as symbolic logic, or **Boolean algebra**.
- Boolean algebra is a branch of mathematics used to describe the manipulation and processing of **binary** information.
- The two-valued Boolean algebra has important application in the design of modern computing systems.
- Boolean algebra is algebra for the manipulation of objects that can take on only **two** values, typically true and false.
- It is common to interpret the digital value **0** as false and the digital value **1** as true.

BOOLEAN ALGEBRA

- Boolean algebra is a form of algebra that deals with single digit binary values and variables.
- Values and variables can indicate some of the following binary pairs of values:
  - ON / OFF
  - TRUE / FALSE
  - HIGH / LOW
  - CLOSED / OPEN
  - 1 / 0
- Three fundamental operators in Boolean algebra
- NOT: unary operator that complements represented as **A'.**
- AND: binary operator which performs logical multiplication i.e. AND ed with would be represented as **A.B**
- OR: binary operator which performs logical addition i.e. OR ed with would be represented as **A+B**

BOOLEAN ALGEBRA PRECEDENCE OF OPERATORS

Boolean expressions must be evaluated with the following order of operator precedence

parentheses

NOT

AND

OR

Example:

$$F = (A(\overline{C + \bar{B}D}) + \overline{B\bar{C}})\bar{E}$$

$$\Downarrow$$

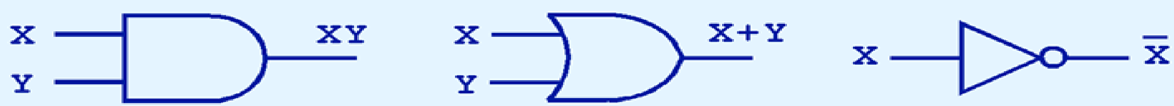
$$F = \underbrace{\underbrace{\left(\underbrace{\underbrace{A \left(\underbrace{\overline{C + \underbrace{\underbrace{\bar{B}}\,D}}} \right)}} + \underbrace{\underbrace{\overline{B\,\underbrace{\bar{C}}}}} \right)}\;\underbrace{\bar{E}}}$$

Logic Gates - Boolean functions are implemented in digital computer circuits called **gates**.

- A gate is an electronic device that produces **a result** based on two or more input values.
  - Gates consist of one to six **transistors**, but digital designers think of them as a single unit.
  - Integrated circuits contain collections of gates suited to a particular purpose.

Symbols for Logic Gates

The three simplest gates are the AND, OR, and NOT gates.



X AND Y

X	Y	XY
0	0	0
0	1	0
1	0	0
1	1	1

X OR Y

X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1

NOT X

X	$\overline{X}$
0	1
1	0

- Another very useful gate is the exclusive OR (XOR) gate.
- The output of the XOR operation is true only when the values of the inputs differ.
-

X XOR Y

X	Y	X ⊕ Y
0	0	0
0	1	1
1	0	1
1	1	0

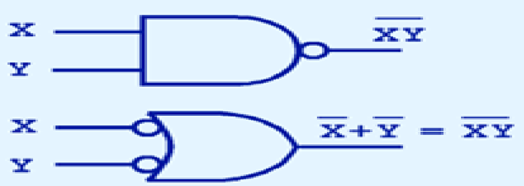
The exclusive OR (XOR) Gate

Universal Gates

- Two other common gates are NAND and NOR, which produce complementary output to AND and OR.

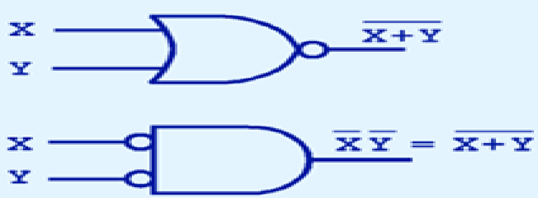
X NAND Y

X	Y	X NAND Y
0	0	1
0	1	1
1	0	1
1	1	0



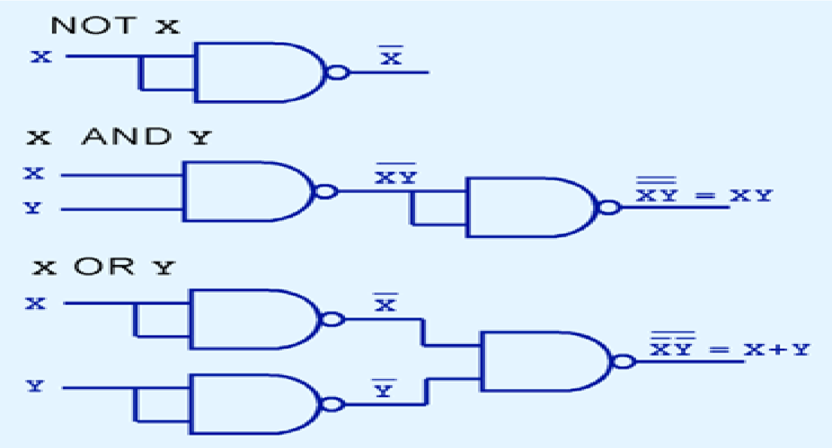
X NOR Y

X	Y	X NOR Y
0	0	1
0	1	0
1	0	0
1	1	0



The Truth Table and Logic Symbols for NAND and NOR Gates

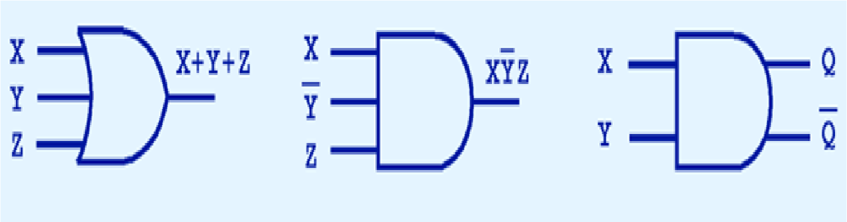
• NAND and NOR are known as universal gates because they are inexpensive to manufacture and any Boolean function can be constructed using only NAND or only NOR gates.



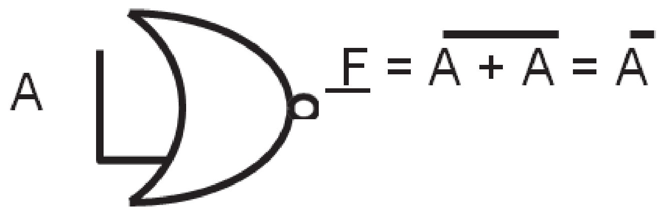
Three Circuits Constructed Using Only NAND Gates

Multiple Input Gates

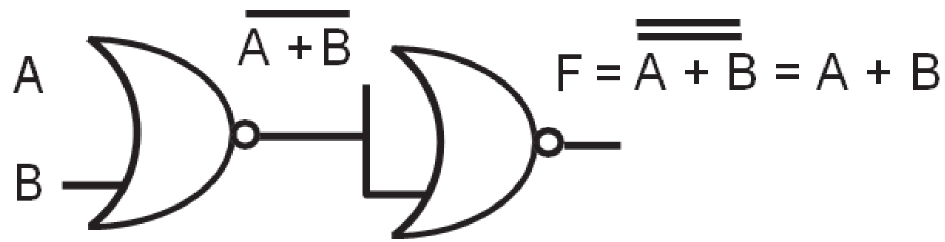
- Gates can have multiple inputs and more than one output.



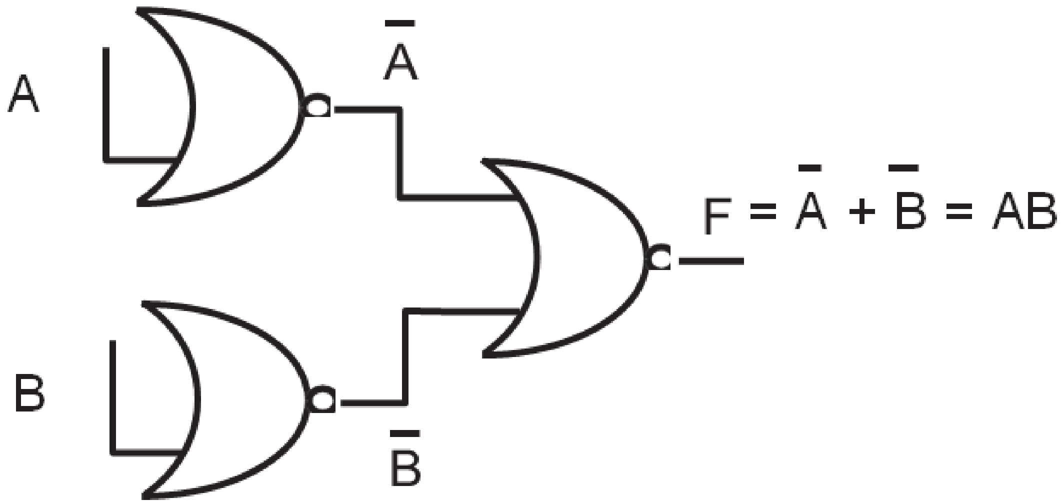
1. "NOT" GATE (INVERTER)



2. "OR" GATE



3. "AND" GATE



BOOLEAN ALGEBRA BASIC IDENTITIES

$X + 0 = X$	$X \cdot 1 = X$	Identity
$X + 1 = 1$	$X \cdot 0 = 0$	
$X + X = X$	$X \cdot X = X$	Idempotent Law
$X + X' = 1$	$X \cdot X' = 0$	Complement
$(X')' = X$		Involution Law
$X + Y = Y + X$	$XY = YX$	Commutativity
$X + (Y + Z) = (X + Y) + Z$	$X(YZ) = (XY)Z$	Associativity
$X(Y + Z) = XY + XZ$	$X + YZ = (X + Y)(X + Z)$	Distributivity
$X + XY = X$	$X(X + Y) = X$	Absorption Law
$X + X'Y = X + Y$	$X(X' + Y) = XY$	Simplification
$(X + Y)' = X'Y'$	$(XY)' = X' + Y'$	DeMorgan's Law
$XY + X'Z + YZ = XY + X'Z$	$(X + Y)(X' + Z)(Y + Z) = (X + Y)(X' + Z)$	Consensus Theorem

A+AB	= A(1+B) = A . 1	(Rule 2)
	= A	(Rule 4)

A + AB =	
= (A + AB) + AB	(Rule 10)
= (AA + AB) + AB	(Rule 7)
= AA +AB + AA ± AB i.e. adding AA =0 = (A+A)(A + B)	(Rule 8) (Factoring)
= 1 . (A + B) = A + B	(Rule 6)

(A + B)(A + C) = = AA + AC + AB + BC	(Distrib.)
= A +AC +AB +BC	(Rule 7)
= A(1+C) + AB + BC	(Distrib.)
= A . 1 + AB + BC	(Rule 4)
= A + AB + BC	(Rule 2)
= A(1 + B) + BC	(Distrib.)
= A .1 + BC	(Rule 2)
= A + BC	(Rule 4)

BOOLEAN ALGEBRA DUALITY PRINCIPLE

- **Principle of Duality :**
  - States that a Boolean equation remains valid if we take the dual of the expressions on both sides of the equals sign.
  - The dual can be found by interchanging the AND and OR operators along with also interchanging the 0's and 1's.
  - This is evident with the duals in the basic identities.
  - For instance: DeMorgan's Law can be expressed in two forms

$$(X + Y)' = X'Y' \quad \text{as well as} \quad (XY)' = X' + Y'$$

$\overline{(xy)} = \bar{x} + \bar{y}$ and $\overline{(x+y)} = \bar{x}\bar{y}$

x	y	(xy)	$\overline{(xy)}$	$\bar{x}$	$\bar{y}$	$\bar{x}+\bar{y}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

1. THIS STATES THAT THE INVERSE (i.e.) OF A PRODUCT [**AND**] IS EQUAL TO THE SUM [**OR**] OF THE COMPLEMENTS
2. THIS STATES THAT THE INVERSE (COMPLEMENT) OF A SUM [**OR**] IS EQUAL TO THE PRODUCT [**AND**] OF THE COMPLEMENTS

TABLE 3.6 Truth Tables for the AND Form of De Morgan's Law

<u>COMPLEMENTS:</u>

F(x, y, z) = x' + yz' and its complement, F'(x, y, z) = x(y' + z)

x	y	z	$y\bar{z}$	$\bar{x}+y\bar{z}$	$\bar{y}+z$	$x(\bar{y}+z)$
0	0	0	0	1	1	0
0	0	1	0	1	1	0
0	1	0	1	1	0	0
0	1	1	0	1	1	0
1	0	0	0	0	1	1
1	0	1	0	0	1	1
1	1	0	1	1	0	0
1	1	1	0	0	1	1

TABLE 3.7 Truth Table Representation for a Function and Its Complement

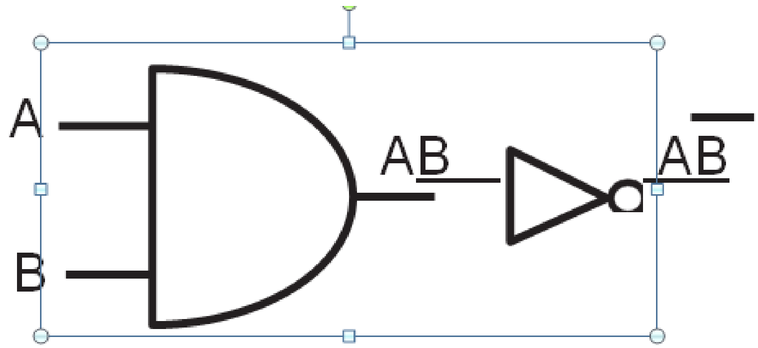
De Morgan's Theorem 1

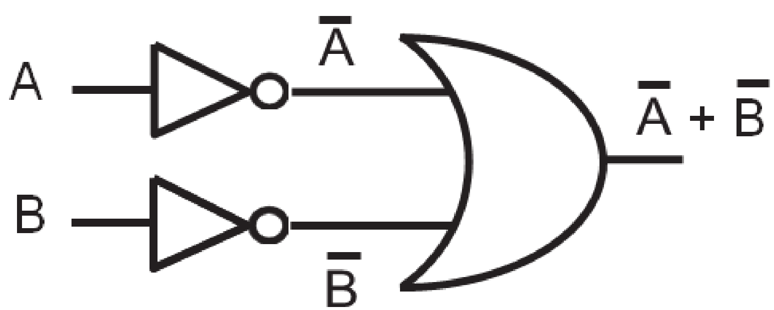
Theorem 1 $\overline{A \cdot B} = \overline{A} + \overline{B}$

Theorem 1 $\overline{A + B} = \overline{A} \cdot \overline{B}$

A	B	$\overline{AB}$	$\overline{A}$	$\overline{B}$	$\overline{A} + \overline{B}$
0	0	1	1	1	1
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	0	0	0

PROOF OF (1): (AB)’ = A’+B’





De Morgan's Theorem 2

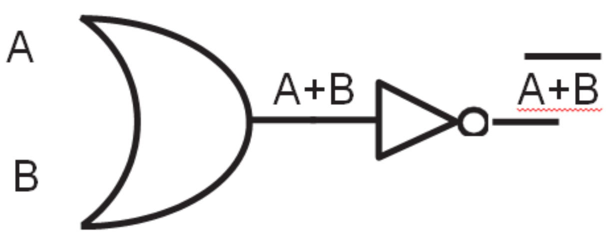
Theorem 2 $\overline{A + B} = \overline{A} \cdot \overline{B}$

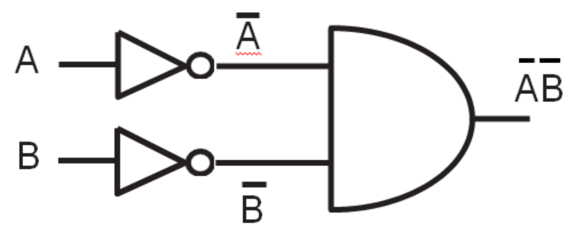
$\overline{A + B} = \overline{A}.\overline{B}$

NOR = Bubbled AND

PROOF OF (2): $\overline{A + B} = \bar{A} \; . \; \bar{B}$

A	B	$\overline{A+B}$	A	B	A+B	$\bar{A} . \bar{B}$
0	0	0	1	1	**1**	**1**
0	1	1	1	0	**0**	**0**
1	0	1	0	1	**0**	**0**
1	1	1	0	0	**0**	**0**





EXAMPLES :

- Simplify the following expression :

$$F = BC + B\overline{C} + BA$$

$$F = B(C + \overline{C}) + BA$$

$$F = B \cdot 1 + BA$$

$$F = B(1 + A)$$

$$F = B$$

$$F = A + \overline{A}B + \overline{A}\overline{B}C + \overline{A}\overline{B}\overline{C}D + \overline{A}\overline{B}\overline{C}\overline{D}E$$

Simplification

$$F = A + \overline{A}(B + \overline{B}C + \overline{B}\overline{C}D + \overline{B}\overline{C}\overline{D}E)$$

$$F = A + B + \overline{B}C + \overline{B}\overline{C}D + \overline{B}\overline{C}\overline{D}E$$

$$F = A + B + \overline{B}(C + \overline{C}D + \overline{C}\overline{D}E)$$

$$F = A + B + C + \overline{C}D + \overline{C}\overline{D}E$$

$$F = A + B + C + \overline{C}(D + \overline{D}E)$$

$$F = A + B + C + D + \overline{D}E$$

$$F = A + B + C + D + E$$

Simplification using Boolean Algebra laws :

$$Z = A'BC + AB'C' + AB'C + ABC' + ABC$$

$$= A'BC + AB'(C' + C) + AB(C' + C)$$

$$= A'BC + AB' + AB$$

$$= A'BC + A(B' + B)$$

$$= A'BC + A$$

$$= BC + A$$

$(X \cdot Y') + Y = X + Y$ with $X = BC$ and $Y = A$

- We can use Boolean identities to simplify the function as follows:

$$F(X,Y,Z) = (X + Y)(X + \overline{Y})\overline{(X\overline{Z})}$$

$(X + Y)(X + \overline{Y})\overline{(X\overline{Z})}$	Idempotent Law (Rewriting)
$(X + Y)(X + \overline{Y})(\overline{X} + Z)$	DeMorgan's Law
$(XX + X\overline{Y} + XY + Y\overline{Y})(\overline{X} + Z)$	Distributive Law
$((X + Y\overline{Y}) + X(Y + \overline{Y}))(\overline{X} + Z)$	Commutative & Distributive Laws
$((X + 0) + X(1))(\overline{X} + Z)$	Inverse Law
$X(\overline{X} + Z)$	Idempotent Law
$X\overline{X} + XZ$	Distributive Law
$0 + XZ$	Inverse Law
XZ	Idempotent Law

Practice Problems

1. AB+(AC)’+AB’C(AB+C)=1
2. ABC+ABC’+AB’C=A(B+C)
3. Y=(A’C[A’BD]’+A’BC’D’+AB’C
4. (A+B)(A’C’+C)(B’+AC)’=A’B
5. If AB’+A’B=C , show that AC’+A’C=B
6. AB+BC+B’C=AB+C
7. A’B+AB+A’B’

THANK YOU