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SNS INSTITUTIONS


## DEPARTMENT OF AEROSPACE ENGINEERING

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| Faculty Name | : | **Dr.M.Subramanian, Prof & Head/ Aerospace** | Academic Year | : | **2024-2025 (Odd)** |
| Year & Branch | : | **III Aerospace** | Semester | : | **V** |
| Course | : | **19ASB302 – Finite Element Method for Aerospace** | | | |

## STIFFNESS MATRIX FOR 1-D BAR ELEMENT

Stiffness matrix $[K]$

$$= \int_v [B]^T [D] [B]\, dv \qquad D = E \text{ (young's modulus)}$$

$$[dv = A \cdot dx]$$

Displacement function $U = N_1 u_1 + N_2 u_2$

$$N_1 = \frac{L - x}{L} \qquad N_2 = \frac{x}{L}$$

$[B]$ strain displacement matrix $= \left[\frac{dN_1}{x} \quad \frac{dN_2}{x}\right]$

$$= \left[-\frac{1}{L} \quad \frac{1}{L}\right]$$

$$[B]^T = \begin{bmatrix} -1/L \\ 1/L \end{bmatrix}$$

Substitute $[B]$, $[D]$, $[B]^T$ values in stiffness matrix equation.

$$K = \int_0^L \begin{bmatrix} -1/L \\ 1/L \end{bmatrix} E \left[-1/L \quad 1/L\right] A\, dx.$$

$$= \int_0^L \begin{bmatrix} \frac{1}{L^2} & -1/L^2 \\ -1/L^2 & 1/L^2 \end{bmatrix} EA\,dx$$

$$= AE \begin{bmatrix} 1/L^2 & -1/L^2 \\ -1/L^2 & 1/L^2 \end{bmatrix} \int_0^L dx$$

$$= AE \begin{bmatrix} 1/L^2 & -1/L^2 \\ -1/L^2 & 1/L^2 \end{bmatrix} \left[x\right]_0^L$$

$$= AE \begin{bmatrix} 1/L^2 & -1/L^2 \\ -1/L^2 & 1/L^2 \end{bmatrix} \left[L-0\right]$$

$$= \frac{AE\,L}{L^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \Rightarrow K = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

The properties of stiffness matrix

* It is symmetric
* The sum of elements in any columns is equal to zero.