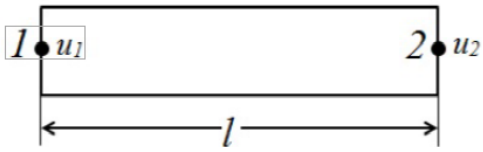
UNIT II- ONE DIMENSIONAL PROBLEMS

DISPLACEMENT FUNCTION:

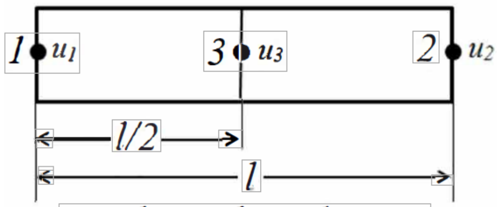
For lD linear bar element



linear bar element

$$u = N_1 u_1 + N_2 u_2 \qquad N_1 = 1 - \frac{x}{l}, N_2 = \frac{x}{l}$$

For lD quadratic element



quadratic bar element

$$u = N_1 u_1 + N_2 u_2 + N_3 u_3$$

$$N_1 = 1 - \frac{3x}{l} + \frac{2x^2}{l^2} \qquad N_2 = -\frac{x}{l} + \frac{2x^2}{l^2} \qquad N_3 = \frac{4x}{l} + \frac{4x^2}{l^2}$$

N_1 Shape Function at node 1 , U_1 Displacement at node 1 , N_2 Shape Function at node 2 , U_2 Displacement at node 2 , N_3 Shape Function at node 3 , U_3 Displacement at node 3

STIFFNESS MATRIX:

For 1D linear bar element $[k] = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

For 1D quadratic element $[k] = \frac{AE}{3L}\begin{bmatrix} 7 & 1 & -8 \\ 1 & 7 & -8 \\ -8 & -8 & 16 \end{bmatrix}$

A-Area of the element mm^2

E-Young's Modulus of the element N/mm^2

L-length of the element

GENERAL FORCE EQUATION:

$\{F\}=[k]\{u\}$ $\{F\}$-***Global Force vector ,*** $[k]$-Global stiffness matrix, $\{u\}$-Global Displacement matrix

REACTION FORCE:

$\{R\}=[k]\{u\}-\{F\}$ $\{R\}$-Reaction force

IF THE BODY is SUBJECTED to SELF -WEIGHT:

For 1D linear bar element $\{F\}=\frac{\rho AL}{2}\begin{Bmatrix}1\\1\end{Bmatrix}$

For lD quadratic element $\{F\}=\rho AL\begin{Bmatrix}\frac{1}{6}\\\frac{1}{6}\\\frac{2}{3}\end{Bmatrix}$

ρ-Unit weight density of the element (N/mm^3), A-Area of the element (mm^2), L-Length of the element (mm).

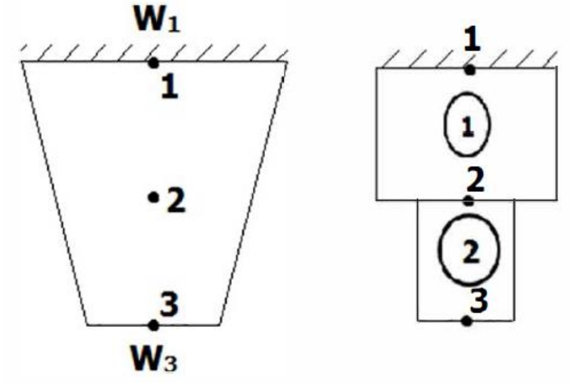
STRESS ON THE ELEMENT:

$$\sigma=E.\frac{du}{dx}$$

If stress on element one should found then the formula will be $\sigma=E.\frac{u_2-u_1}{L_1}$

E -Young's Modulus of the element N/mm^2 , L_1- Length of the element one mm , u_1 Displacement at nodal point 1 mm, u_2 Displacement at nodal point 2

FOR TAPER PLATE:



For rectangular cross section *Area= Width* Thickness*

At any point of x $$A_x = A_1 - (A_1 - A_3)\frac{x}{l}$$

Area at node 1: $$A_1 = W_1 \times t_1$$

Area at node 2(Applicable only for mid - point): $A_2 = W_2 \times t_2$

$$A_2 = \left(\frac{W_1 + W_3}{2}\right) \times t_2$$

Area at node 3: $$A_3 = W_3 \times t_3$$

Average area of element 1: $A^1 = \dfrac{A_1 + A_2}{2}$

Average area of element 2: $A^2 = \dfrac{A_2 + A_3}{2}$

For circular cross section $$Area = \frac{\pi}{4} d^2$$

TEMPERATURE EFFECTS ON STRUCTURAL PROBLEM:

STIFFNESS MATRIX: $[k]=\frac{AE}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$

A-Area of the element (mm^2 ***), E-Young's Modulus of the element(*** N/mm^2), ***L-Length of the element(mm)***

THERMAL LOAD: $\{F\}=EA\alpha\Delta T\begin{Bmatrix}-1\\ 1\end{Bmatrix}$

A-Area of the element (mm^2 ***), E-Young's Modulus of the element(*** N/mm^2),
α - Coefficient of thermal expansion ($/^oC$), ΔT - Temperature difference

THERMAL STRESS:

$\{\sigma\}=E\frac{du}{dx}-E\alpha\Delta T$ $\quad$ $E\alpha\Delta T$ - Thermal Strain

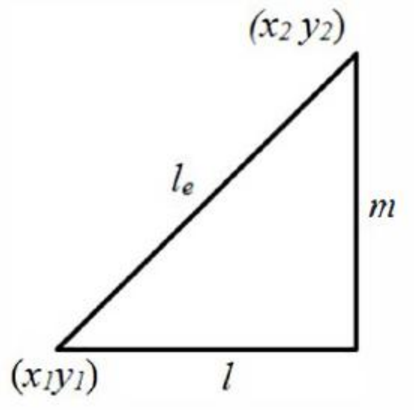
For element 1 $\{\sigma\}=E_1\frac{u_2-u_1}{L_1}-E_1\alpha_1\Delta T$

REACTION FORCE: $\{R\}=[k]\{u\}-\{F\}$

$\{R\}$-Reaction force , $\{F\}$-***Force vector (Global),*** $[k]$-Global stiffness matrix,
$\{u\}$-Global Displacement matrix

SPRINGS: STIFFNESS MATRIX $[k]=k^s\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ k^s - Stiffness of the spring(N/mm)

TRUSSES:



STIFFNESS MATRIX:

$$[k]=\frac{E_eA_e}{l_e}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

A_e -Area of the element (mm^2), E_e - *Young's Modulus of the element* (N/mm^2),
l_e - Length of the element (mm)

$$l_1=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Length of the element 1 $l=\frac{x_2-x_1}{l_1}$ $\qquad m=\frac{y_2-y_1}{l_1}$

STRESS:

Stress of element 1 $\qquad \sigma_1=\frac{E}{l_1}[-l_1 \quad -m_1 \quad l_1 \quad m_1]\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$

u matrix varies according the element

BEAMS: DISPLACEMENT FUNCTION:

$$u = N_1 u_1 + N_2 u_2 + N_3 u_3 + N_4 u_4$$

$$N_1 = \frac{1}{L^3}(2x^3 - 3x^2 L + L^3) \qquad N_2 = \frac{1}{L^3}(x^3 L - 2x^2 L^2 + xL^3)$$

$$N_3 = \frac{1}{L^3}(-2x^3 + 3x^2 L) \qquad N_4 = \frac{1}{L^3}(x^3 L - x^2 L^2)$$

N_1 Shape Function at node 1,U_1 Displacement at node 1 , N_2 Shape Function at node 2 , U_2 Displacement at node 2 ,N_3 Shape Function at node 3 , U_3 Displacement at node 3 , N_4 Shape Function at node 4 , U_4 Displacement at node 4

STIFFNESS MATRIX:

$$k = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

I-Moment of Inertia (mm^4), E- Young's Modulus(N/mm^2), L-Length of the beam(mm)